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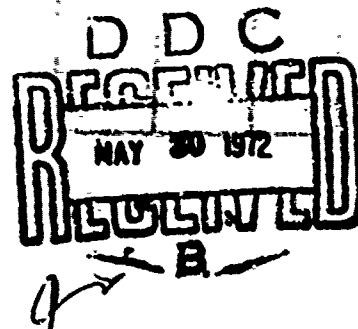
Report No. 591  
Job No. 13,075

DESIGN STUDY OF DAMPING TECHNIQUES

Quarterly Progress Report  
July-September 1958

BuShips Contract NObS-72452,  
Index No. NS-13-212

20 October 1958



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## DESIGN STUDY OF DAMPING TECHNIQUES

### I. SCOPE OF STUDY

Meetings were held with representatives of BuShips Code 375 on 30 June and 27 August at which the scope of this project was discussed. It was agreed that the studies under the present contract should cover the following three phases:

Phase I - Damping by Single Homogeneous Layers and by Simple Constrained Layers.

- A. Analytical studies of damping on large plates, neglecting the effects of boundaries.
- B. Analytical studies of the limitations of the results of Part A, including:
  - 1. effects of boundaries
  - 2. limits on thickness of application
  - 3. limits of temperature and frequency
  - 4. effects of non-linearities
  - 5. effects of water load
  - 6. effects of plate curvature

### C. Experimental Studies

Work on Part A of this phase is described in the present report. Work on Parts B and C is only now getting underway.

Phase II - Extensions of Analysis to Fibrous Damping Layers and to Stiff Damping Layers.

- A. An exploratory analysis of the constrained fibrous damping layer aimed at understanding the mechanism of damping by constrained fibrous damping layers.
- B. Analytical study of constrained-layer damping when the damping layer is relatively stiff.

It is planned to initiate work on both parts of this phase during the October-December quarter.

Phase III - Study of Combination Damping Treatments

- A. Two homogeneous damping layers
- or B. A constrained damping layer with a homogeneous layer applied over it.

It was agreed that the decision as to which parts of Phase III would be undertaken would be postponed until work in Phase II was well underway.

During this quarter, a memorandum was submitted by the contractor outlining work that has been and is being undertaken in the field of vibration damping for a number of sponsors. It was indicated that although these studies each cover different damping treatments and/or are intended for different applications, each should benefit from knowledge gained through the others.

## II. REPORT ON PHASE I-A: ANALYTICAL STUDIES OF THE DAMPING OF LARGE PLATES BY SIMPLE VISCO-ELASTIC DAMPING TREATMENTS

### Introduction

There are currently two distinct methods of damping the flexural vibrations of plates with visco-elastic materials. The older and more common method involves the application of a relatively thick layer of damping material. An example of this is automobile undercoat. The equations governing this type of damping were derived independently by Oberst and Liénard<sup>1-4/</sup>. The damping mechanism involves the internal losses of the visco-elastic material undergoing alternating extensional strains. The damping achieved is roughly proportional to the product of the resistive component of the Young's modulus\* and the square of the thickness of the damping layer.

The second type of application for the damping of plate vibrations, called a "damping tape" or "constrained damping layer", has been developed recently in the aircraft industry. It consists of a relatively thin layer of visco-elastic material and a stiff covering "foil". The damping achieved by such constrained damping layers involves the shear motion of the visco-elastic material between the two stiff media. The damping achievable increases with increased stiffness of the constraining layer. There is an optimum

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\* This component is sometimes called the "loss modulus" to distinguish it from the elastic component or "storage modulus".

thickness of the damping material, which may be quite small, and which is a function of the temperature and frequency range for which maximum damping is desired. The equations governing the damping by constrained damping layers were recently derived by Kerwin<sup>5,6/</sup>.

A more general analysis, in which the constrained-damping-layer configuration is treated as a special case, has been carried out by BBN and presented as a report to the Office of Naval Research<sup>7/</sup>. In the past quarter, we have taken the resultant equations and have developed charts whereby the loss factor may be calculated from knowledge of the geometry of the configuration and of the elastic moduli of the materials. We have then presented these results in the form of charts useful in understanding damping results and in designing damping treatments.

#### Significance of the Analyses

The analyses of the damping of flexural vibrations by visco-elastic damping layers given in this Progress Report are extensions of the analyses reported in Ref 7. As such, they are limited to large, flat plates for which the effects of boundaries can be neglected. None of the secondary effects that might influence the actual performance of damping treatments on submarine hulls have yet been included. Specifically, the effect of plate curvature, external pressure and of water load have not yet been considered. Nor have the practical limitations of material properties been taken into account.

Despite these limitations, we believe that the analyses have appreciable practical value. The results are indicative of the maximum damping that can be achieved by these mechanisms. The calculations are also useful in comparing various damping mechanisms and in scaling from one structure to another. It is to be expected that many of the secondary effects, such as water load and plate curvature, will have similar effects on different damping treatments. Thus, many comparisons made for flat plates will also apply qualitatively to submarine structures.

No experimental confirmations of the analyses are given in the present report. We feel that the experimental confirmations already reported in Refs 5 and 7 have established confidence in the basic theories involved.

#### Homogeneous Damping Layers

The equations governing the damping of homogeneous visco-elastic applications are well known, having been derived and rederived many times (See, for example, Refs 1, 3 and 7). An approximate relation, which holds over much of the useful range is:

$$\eta \doteq 14 \frac{\eta_2 E_2}{E_1} \left( \frac{H_2}{H_1} \right)^2 \quad (1)$$

where:

$\eta$  is the loss factor\*

$E$  is Young's modulus

$H$  is thickness

subscript 1 refers to the base plate

subscript 2 refers to the added damping layer

\* The relationships of  $\eta$  to other measures of damping are covered in Appendix B.

This equation can be expressed in terms of the weights,  $W$ , per unit area of the material through the relation:

$$W = \gamma H \quad (2)$$

where  $\gamma$  is the weight per unit volume.

The resultant expression is:

$$\eta \doteq 14 \left( \frac{\eta_2 E_2}{E_1} \right) \left( \frac{\gamma_1}{\gamma_2} \right)^2 \left( \frac{W_2}{W_1} \right)^2 \quad (3)$$

There is a wide variety of homogeneous-layer damping materials, having best damping at different temperatures or having different mechanical properties, e.g., adhering, drying, etc. We may estimate the damping achievable with the better products when applied to steel plate by taking the best combination of parameters that we have found for available materials<sup>2/</sup>. These are:

$$\eta_2 E_2 = 4 \times 10^9 \text{ dynes/cm}^2$$

$$\gamma_2 = 38.5 \text{ lbs/ft}^3$$

$$E_1 = 2 \times 10^{12} \text{ dynes/cm}^2$$

$$\gamma_1 = 480 \text{ lbs/ft}^3$$



The resultant expression for the maximum damping factor is:

$$\eta_{\max} = 4 \left( \frac{W_2}{W_1} \right)^2 \quad (4)$$

Although the properties of the various compounds vary widely, the general tendency is to have a higher resistive stiffness (loss modulus)  $\eta E$  associated with denser materials, so that the coefficient in Eq (4) will tend to remain constant\*. Using Eq (4), we have constructed Fig 1 to show how the approximate maximum damping varies with the weight of the application relative to that of the plate. In Fig 2, the maximum damping is given as a function of the weight of the treatment for the range of plate thicknesses likely to be encountered in submarine construction.

The important conclusion from the foregoing analysis is that, although the relative weight of the damping material is not a fundamental variable in the damping equations, for practical purposes it is the controlling factor in determining the amount of damping that can be achieved with homogeneous-layer damping treatments.

#### Constrained Damping Layers

When a thin layer of damping material is constrained by a stiffener layer, the resultant damping is almost entirely attributable to the shear motion of the damping layer. Analyses of this

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\* If anything, the coefficient will be somewhat lower with denser materials.

mechanism are to be found in References 5 and 7. Because this theory may not be familiar to some of the readers of this report, a brief version is presented in Appendix A. The final theoretical equation of Reference 7 is not in a form convenient for calculations; but, it is possible to rearrange the terms and obtain an equation more suitable for numerical work. This has been done in Appendix A. The resultant equation is:

$$\eta = \frac{A\beta\Gamma}{1 + C\Gamma + D(1+\beta^2)\Gamma^2} \quad (5)$$

where:  $\beta$  is the loss factor of the shear modulus of the damping material. ( $G_2^* = G_2(1+j\beta)$ ).

$\Gamma$  is the basic parameter governing the shear damping.

$$(\Gamma \doteq \frac{720}{f} \frac{G_2}{E_1 H_2} \text{ for steel or aluminum plates.})$$

A, C and D are functions of relative thicknesses of the foil, damping layer and base plate and of the materials of the foil and base. They are independent of the properties of the damping layer.

The Appendix also contains charts whereby A, C, D and  $\Gamma$  may be calculated from the material properties and dimensions of the layers. Other charts then allow the calculation of the resultant damping factor. Finally, a sample calculation is carried through to illustrate the use of the charts.

An important result of the analysis of the shear damping mechanism is the dependence on the properties of the damping layer as given by the shear parameter,  $\Gamma$ . For low values of this parameter, corresponding to a damping layer with little resistance to shear motion, the damping is relatively low. As the layer becomes stiffer, and  $\Gamma$  increases, the damping increases. The damping becomes a maximum at a moderate value of  $\Gamma$  (say,  $0.01 < \Gamma < 0.1$ ), and then decreases with further increase of  $\Gamma$ , as the layer becomes "too stiff". The peak of the curve is relatively broad, the loss factor dropping to one-half its maximum value for  $\Gamma$  about two octaves either side of the optimum.

The shear parameter is a complicated function of frequency and temperature, depending as it does on the shear modulus of the damping material, which is a function of both frequency and temperature. In addition,  $\Gamma$  depends on the thickness of the damping layer. For given operating temperatures and frequencies it is usually possible to select a damping material and/or layer thickness that will yield close-to-maximum damping. When this is done, the damping achieved is primarily a function of the stiffness of the stiffener layer relative to that of the base plate, and of the loss factor,  $\beta$ , of the damping material.

To estimate the damping achievable on flat plates by constrained-layer damping treatments, we have assumed that the proper choice of damping material has been made to give optimum damping with a layer thickness of about 0.030 inches. (This layer thickness was chosen as the minimum that can safely be applied to heavy steel plates without requiring better than normal surface finishes.) We have also assumed steel as the stiffener material, although

aluminum would be slightly more effective on an equal weight basis. Assuming values of  $\beta$  of 0.5, 1 and 1.5, we have plotted the damping factor as a function of the weight of the treatment and the thickness of the stiffener layer. The results are presented in Figs 3-6 for four typical thicknesses of steel plate: 1/4, 1/2, 1 and 2 inches. The three curves for the three values of  $\beta$  may be interpreted as "readily attainable", "probable" and "possible" respectively.

#### Comparison of the Two Types of Damping

The damping achievable by homogeneous damping layers depends roughly on the square of the weight of the material, while that for constrained layers depends approximately linearly on weight. When we compare the curves of Fig 2 with the corresponding curves of Figs 3-6, we may conclude that the two types are likely to give equal values of maximum damping for weights between 10 and 20 percent that of the base plate.

Below 10 percent, the constrained-layer damping will probably be the more effective; while, in cases where more than 20 percent of the plate weight can be devoted to damping, then the homogeneous application should be more effective.

An important corollary of this result is that different types of damping treatments cannot be directly rank ordered if they are tested at ratios of treatment to plate weights different from those which will ultimately be used. An understanding of each damping mechanism is necessary before meaningful comparisons can be made.

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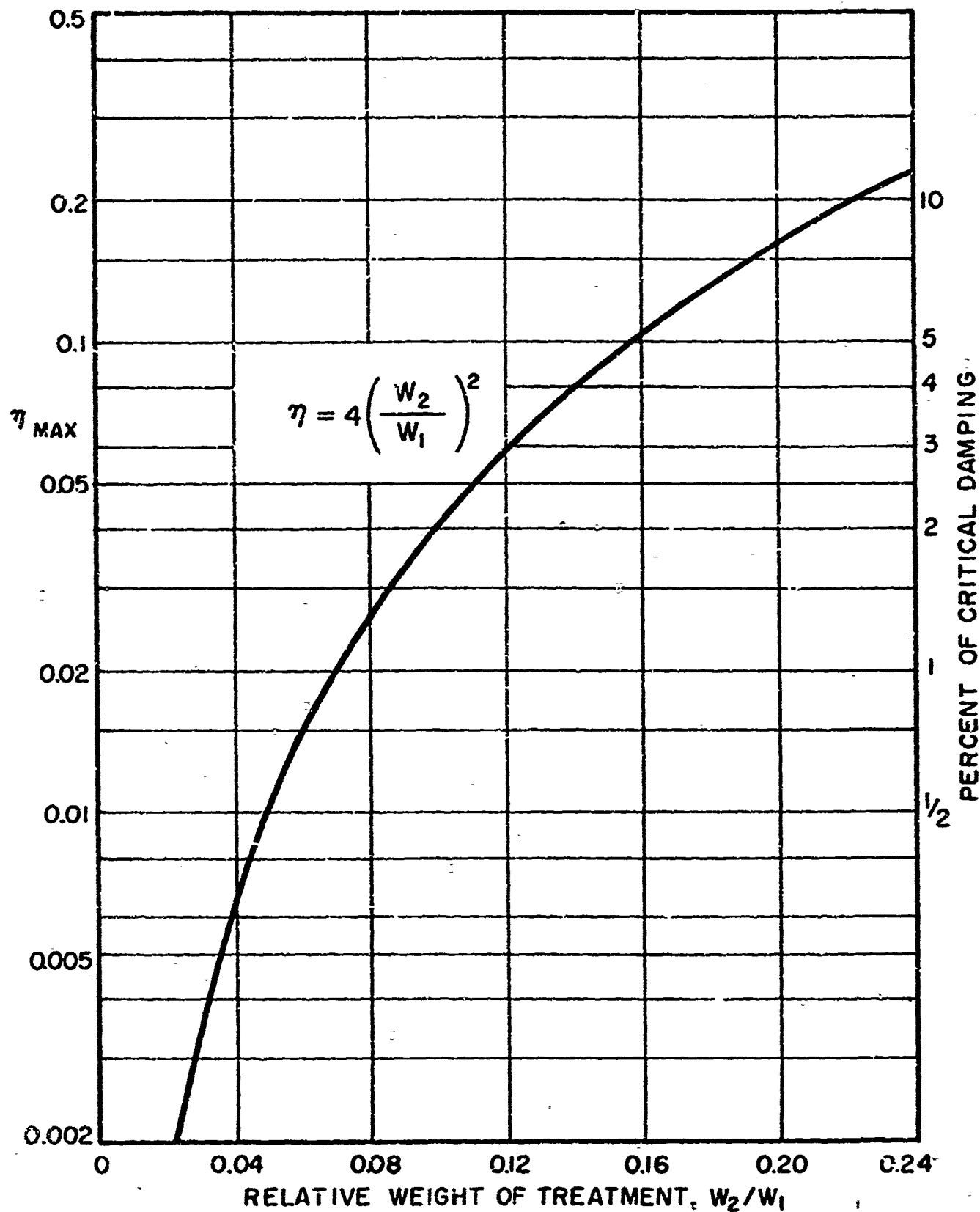


FIG. 1 MAXIMUM DAMPING OF HOMOGENEOUS LAYERS

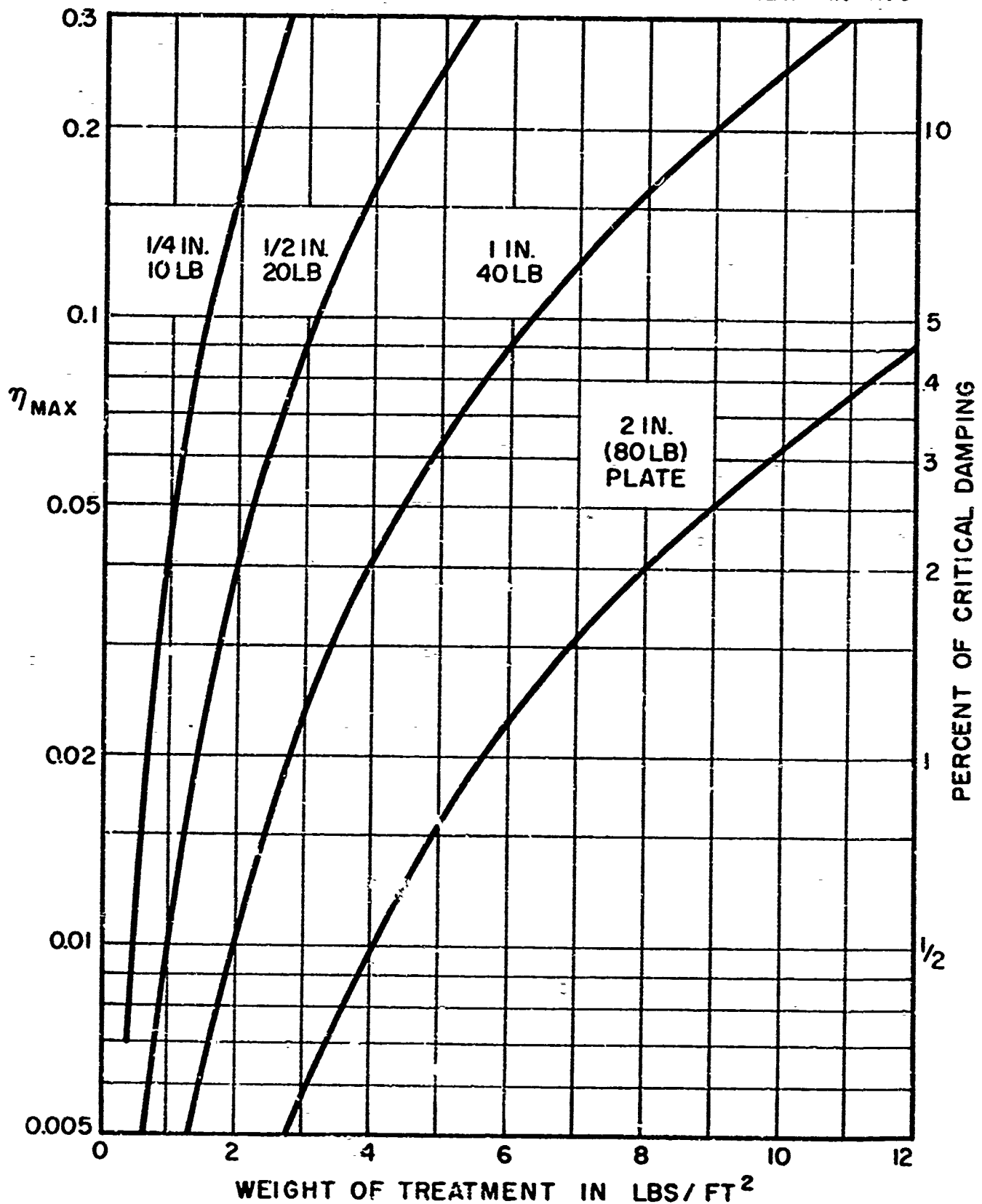


FIG. 2 HOMOGENEOUS DAMPING TREATMENTS  
FOR FOUR PLATE THICKNESSES



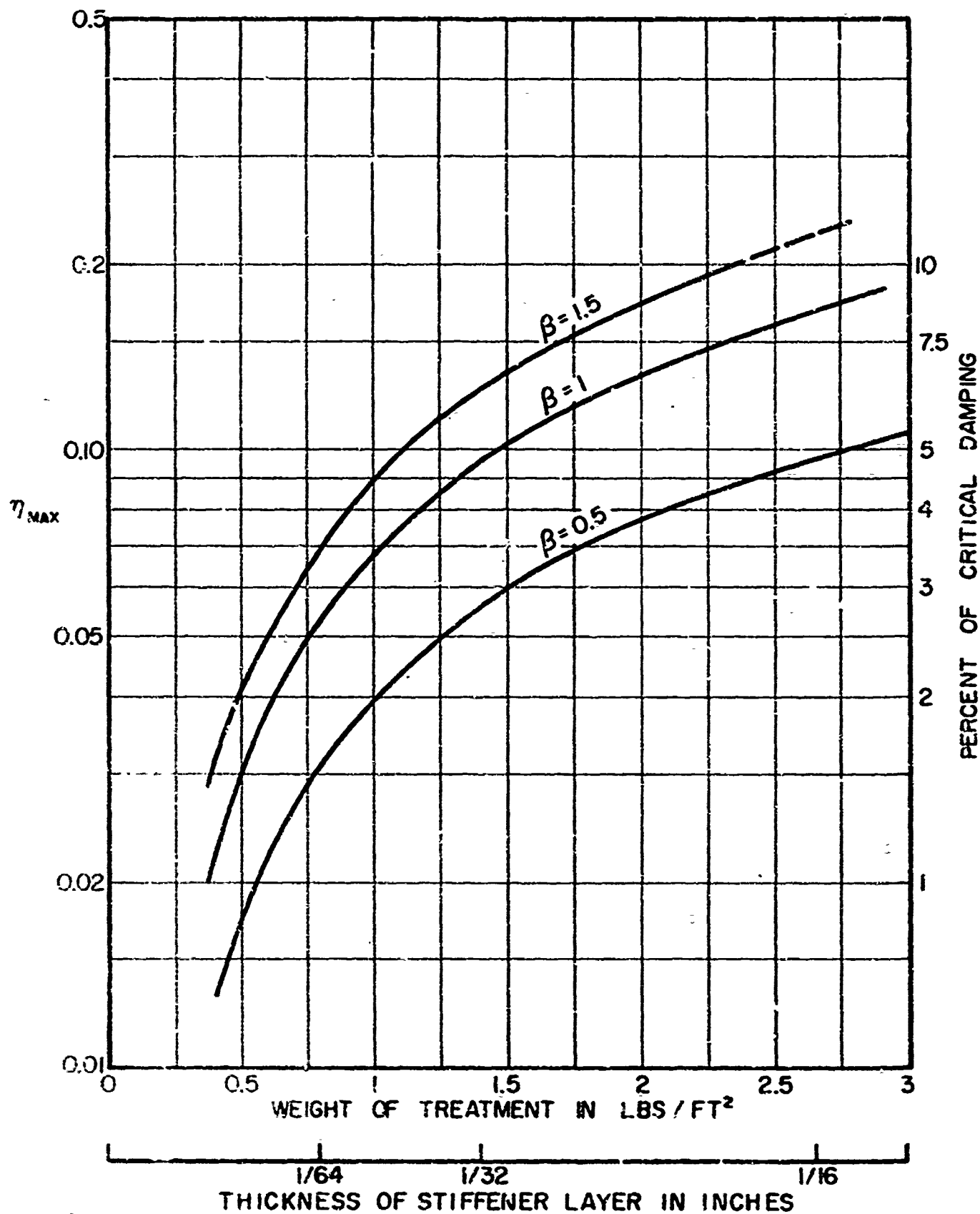


FIG. 3 DAMPING OF 1/4-INCH STEEL PLATES  
(10-POUND PLATE)

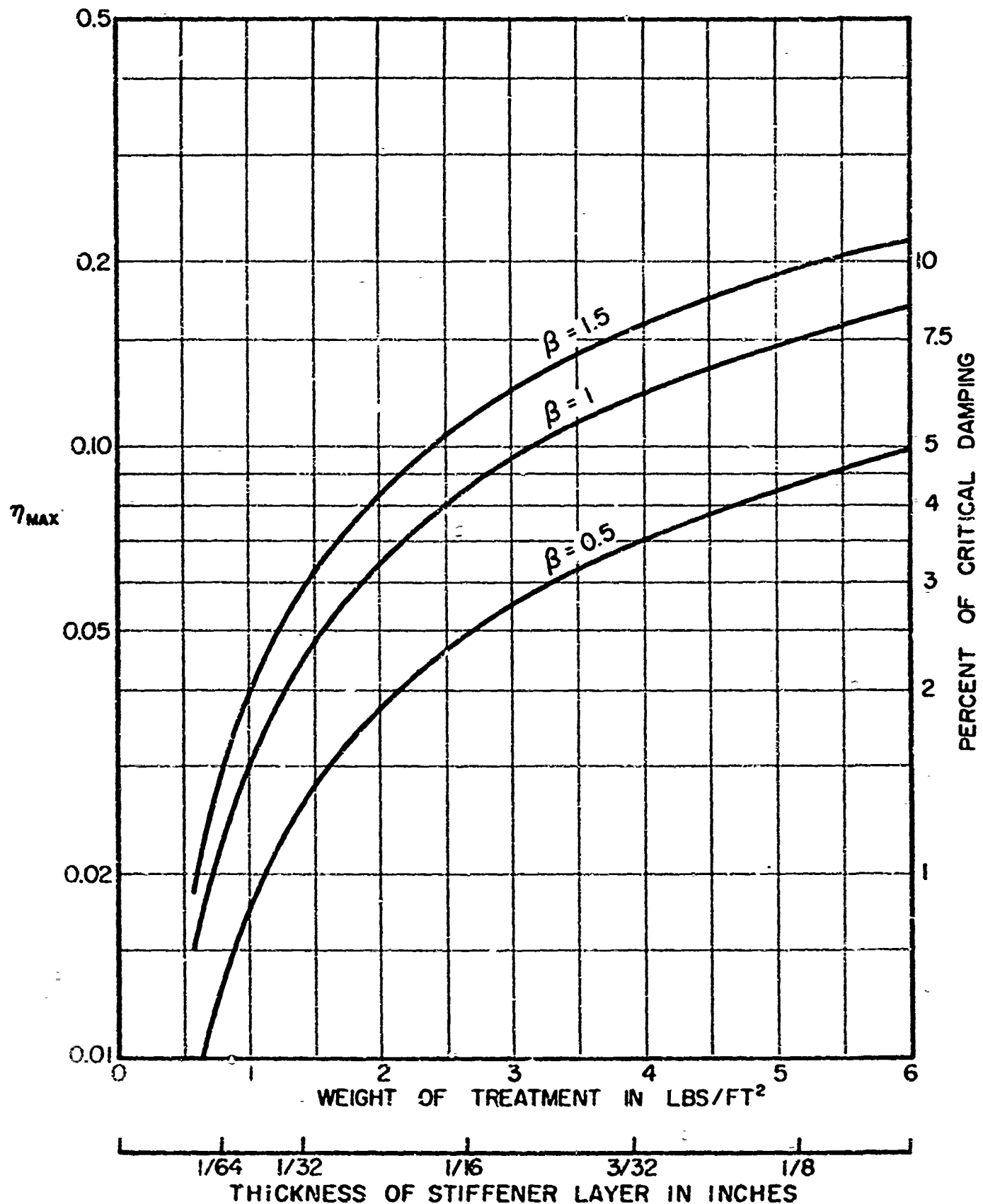


FIG. 4 DAMPING OF 1/2-INCH STEEL PLATES  
(20-POUND FLATE)

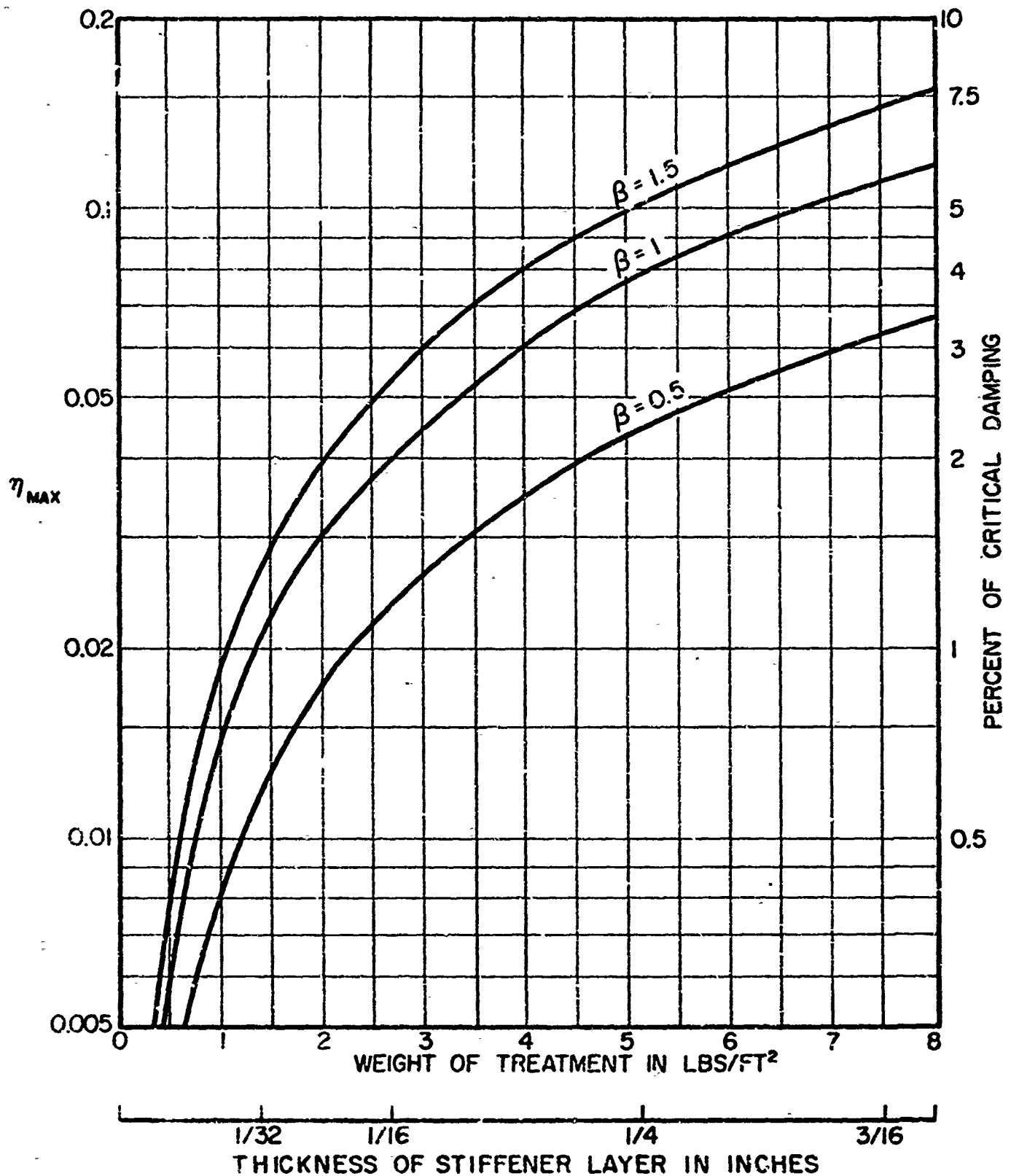


FIG. 5 DAMPING OF 1-INCH STEEL PLATES  
(40-POUND PLATE)

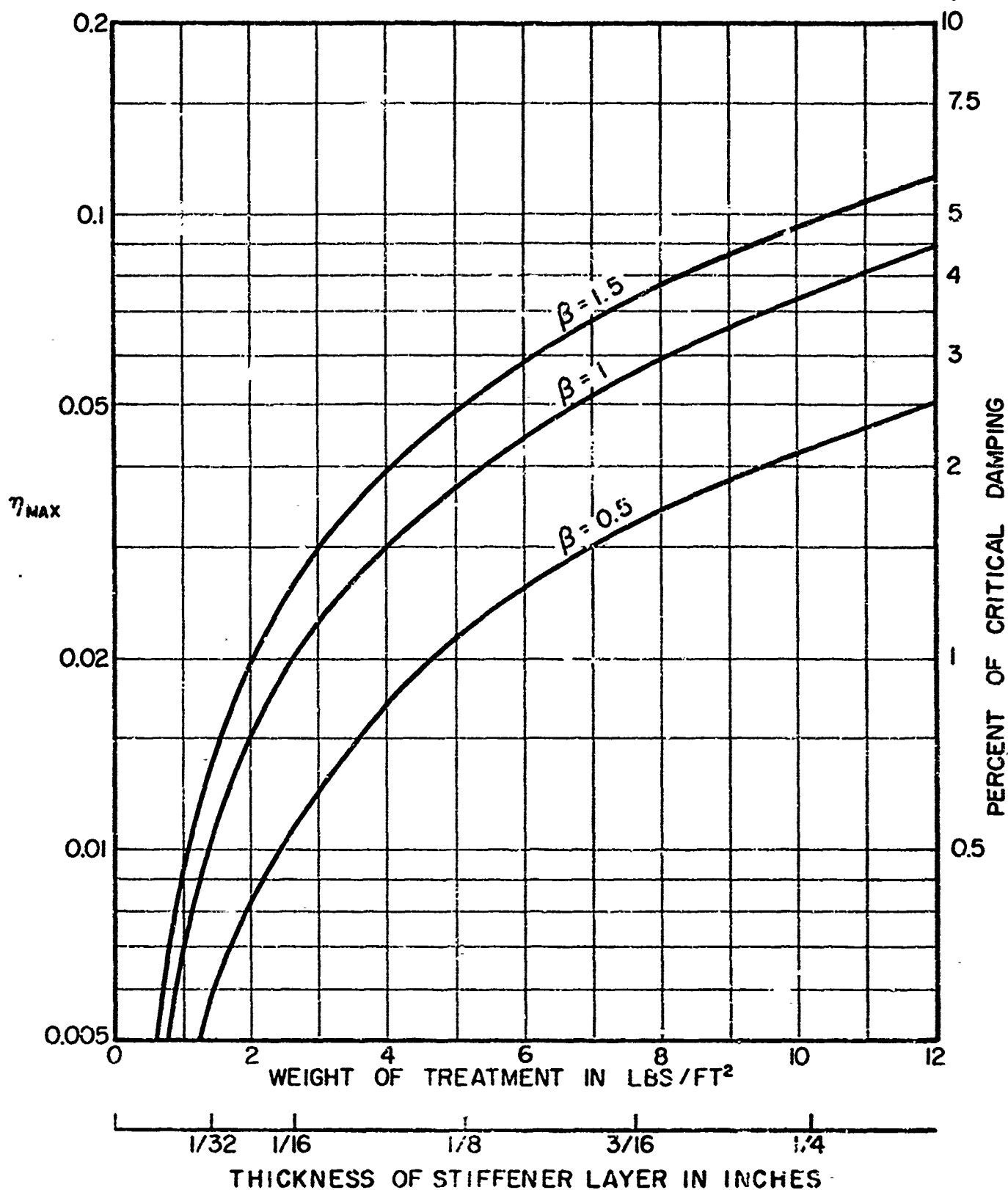


FIG. 6 DAMPING OF 2-INCH STEEL PLATES  
(80-POUND PLATE)

## APPENDIX A

## THE DAMPING OF FLEXURAL VIBRATIONS BY CONSTRAINED DAMPING LAYERS\*

Basic Assumptions

The equations that are presented in this report concern the damping attributable to the shear motion of a thin visco-elastic layer between two stiff plates. Other motions could occur and produce damping, but these are considered negligible relative to the shear motion. In practice, this means that the thicknesses of the plates and of the damping layer are all small compared to the shortest wavelengths of any type of disturbance. As all types of wave motion are ignored, no damping factor is associated with the mass per unit length and the damping factor for the composite plate is simply that of the bending rigidity. All effects of plate curvature and of water load are ignored in this elementary analysis.

Flexural Rigidity of Composite Plate

Figure A-1 represents an element of a three-layer structure in flexure, showing the shear displacement of the middle layer. Layer 1 represents the original undamped plate, the second layer is the damping material, and the third layer the stiff constraining layer. The angle  $\phi$  is the flexural angle of the element and the shear strain,  $\psi$ , is measured relative to  $\phi$  in the opposite direction. The x-direction is chosen as the direction of propagation of the straight-crested flexural wave. It is assumed that all of the layers vibrate in phase and that the flexural angle of an element of foil is the same as that for the corresponding element of the base.

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\*The equations are derived in more detail in Ref 7. Some of the figures used here come from that report.

Figure 2 shows the various thicknesses and distances used in the analysis. The displacement of the neutral plane is represented by  $D$ .

The distribution of extensional strain and stress for the three-layer element is illustrated in Fig 3. It is assumed that the stiffness of the damping layer is small compared to that of the other two layers, so that the net extensional stress of the damping layer can be neglected. As the net force on the composite element is zero, the individual layer forces of the top and bottom layers are equal in magnitude and opposite in direction. Letting the force on the constraining layer be  $F$ , we may write:

$$F = K_3(H_{31}-D) \frac{\partial \phi}{\partial x} - K_3 H_2 \frac{\partial \psi}{\partial x} = K_1 D \frac{\partial \phi}{\partial x} \quad (A-1)$$

where  $K_1$  is the stiffness of a unit area of the 1<sup>th</sup> layer.

The bending moment about the neutral plane of the composite structure is the sum of the moments of the individual layers about their own centers and of the products of the force on each layer by the distance of the center of that layer from the neutral plane of the composite. The expression for the bending rigidity is thus:

$$B = \frac{M}{\partial^2 \phi / \partial x^2} = \frac{1}{12} K_1 H_1^3 + \frac{1}{12} K_3 H_3^3 + K_1 D H_{31} \quad (A-2)$$

The problem now reduces to finding the displacement of the neutral plane of the composite relative to that of the base. Solving Eq (A-1) for  $D$ :

A-2

$$D = \frac{K_3}{K_1 + K_3} \left( H_{31} - H_2 \frac{\partial \psi}{\partial \phi} \right) \quad (A-3)$$

it is seen that this displacement is a function of the relative strain of the damping layer.

The shear strain of the middle layer is related to the shear stress and the shear modulus. As shown in Fig 4, the shear force is proportional to the space-derivative of the force experienced by the foil:

$$\psi = \frac{-1}{G_2} \frac{\partial F}{\partial x} \quad (A-4)$$

where  $G_2$  is the shear modulus of the damping layer. Except near the edges of the structure, the vibratory motion is characterized by expressions of the form:

$$e^{+jp_n x} e^{j\omega_n t} \quad (A-5)$$

where  $p_n$  is the wave-number of the  $n^{\text{th}}$  mode and  $\omega_n$  is the corresponding angular frequency. It follows that the second-derivative of the shear strain is related to the strain itself by:

$$\frac{\partial^2 \psi}{\partial x^2} = -p_n^2 \psi \quad (A-5)$$

and it follows that

$$H_2 \frac{\partial \psi}{\partial \phi} = H_2 \frac{(\partial^2 \psi / \partial x^2)}{(\partial^2 \phi / \partial x^2)} = \left( \frac{K_1 H_2}{G_2} p^2 \right) D \quad (A-7)$$

The quantity in parentheses is a dimensionless coefficient that expresses the properties of the shear layer relative to the stiffness of the basic structure and the wave number of the vibration. This is a fundamental parameter of the damping tape problem. Putting

$$\Gamma = \frac{G_2}{K_1 H_2 p^2} \quad (A-8)$$

substituting Eq (A-7) in Eq (A-3), solving for the displacement  $D$  and then using Eq (A-2), the final expression for the bending rigidity is:

$$B = \frac{1}{12} K_1 H_1^2 + \frac{1}{12} K_3 H_3^2 + \frac{\Gamma K_1 K_3 H_{31}^2}{K_3 + \Gamma(K_1 + K_3)} \quad (A-9)$$

The dissipative properties of the middle layer are considered by expressing the shear modulus as a complex quantity:

$$G_2^* = G_2(1 + j\beta) \quad (A-10)$$

where both  $G_2$  and  $\beta$  are functions of temperature and frequency. Substituting Eq (A-10) for  $G^*$  into Eq (A-8) for  $\Gamma^*$  and then solving Eq (A-9) for  $B^*$  one finds:



$$B^* = \frac{1}{12} K_1 H_1^2 + \frac{1}{12} K_3 H_3^2 + \frac{\Gamma K_1 H_{31}^2 \left[ 1 + \Gamma \left( \frac{K_1 + K_3}{K_3} \right) (1 + \beta^2) \right] + j \beta \Gamma K_1 H_{31}^2}{1 + 2\Gamma \left( \frac{K_1 + K_3}{K_3} \right) + \Gamma^2 \left( \frac{K_1 + K_3}{K_3} \right)^2 (1 + \beta^2)} \quad (A-11)$$

The loss factor for the composite structure is:

$$\eta =$$

$$\frac{\beta \Gamma K_1 H_{31}^2}{\left[ \frac{1}{12} K_1 H_1^2 + \frac{1}{12} K_3 H_3^2 \right] \left[ 1 + 2\Gamma \left( \frac{K_1 + K_3}{K_3} \right) + \Gamma^2 \left( \frac{K_1 + K_3}{K_3} \right)^2 (1 + \beta^2) \right] + \Gamma K_1 H_{31}^2 \left[ 1 + \Gamma \left( \frac{K_1 + K_3}{K_3} \right) (1 + \beta^2) \right]} \quad (A-12)$$

#### Form for Calculations

Equation (A-12) for the loss factor of a plate with constrained layer damping can be rearranged to facilitate calculations. The major independent variable is the shear parameter  $\Gamma$ , while the relative thicknesses of the damping layer and of the constraining layer, as well as the stiffness of the "foil" and the loss factor of the damping material, are parameters. First, all properties of the added layers can be normalized by dividing by the corresponding property of the base plate:

$$h_3 \equiv H_3/H_1 \quad (A-13)$$

$$k_3 \equiv K_3/K_1 = \frac{E_3 H_3}{E_1 H_1} = e_3 h_3 \quad (A-14)$$

The distance between the centers of the constraining layer and the base plate is normalized by dividing by the thickness of the base plate:

$$h_{31} \equiv \frac{H_{31}}{H_1} = \frac{\frac{1}{2} H_1 + H_2 + \frac{1}{2} H_3}{H_1} = \frac{1}{2} (1 + 2h_2 + h_3) \quad (A-15)$$

This distance appears several places in the calculations, always in combination with another term; so, we define:

$$A \equiv \frac{12h_{31}^2}{1 + k_3 h_3^2} = \frac{3(1 + 2h_2 + h_3)^2}{1 + e_3 h_3^2} \quad (A-16)$$

For reasons which will become clear later, we define two additional coefficients that are dependent only on  $h_2$ ,  $h_3$  and  $e_3$  (or  $k_3$ ):

$$C \equiv A + 2 \left( \frac{1 + k_3}{k_3} \right) \quad (A-17)$$

$$D \equiv \left( \frac{1 + k_3}{k_3} \right)^2 \left[ 1 + A \left( \frac{k_3}{1 + k_3} \right) \right] \quad (A-18)$$

With these two additional abbreviations, the equation for the loss factor reduces to:

$$\eta = \frac{\beta A \Gamma}{1 + C\Gamma + D(1 + \beta^2) \Gamma^2} \quad (A-19)$$

which is a form that is suitable for basic calculations.

Optimum Damping

The loss factor for a plate with a constrained damping layer treatment is highly dependent on the shear parameter of the damping layer. At low values of  $\Gamma$  it is proportional to  $\Gamma$ , while at high values it varies inversely with  $\Gamma$ . There is a value of  $\Gamma$  that gives maximum damping. If we call this value  $\Gamma_{opt}$ , we may solve for it by taking the derivative of Eq (A-19) and setting this equal to zero. The result is:

$$\Gamma_{opt} = \frac{1}{\sqrt{D(1 + \beta^2)}} \quad (A-20)$$

Equation (A-19) for the loss factor can now be put into its most simple form by defining three reduced coefficients:

$$A' = \frac{A}{2\sqrt{D}} \quad (A-21)$$

$$C' = \frac{C}{2\sqrt{D}} \quad (A-22)$$

$$\beta' = \tan^{-1} \beta \quad (A-23)$$

The resultant expression for  $\eta$  is:

$$\eta = \frac{2A' \sin \beta' (\Gamma/\Gamma_{opt})}{1 + 2C' \cos \beta' (\Gamma/\Gamma_{opt}) + (\Gamma/\Gamma_{opt})^2} \quad (A-24)$$

where

$$\Gamma_{\text{opt}} = \frac{1}{\sqrt{D}} \cos \beta' \quad (\text{A-25})$$

The maximum damping achievable with a given damping treatment is given by:

$$\eta_{\text{max}} = A' \frac{\sin \beta'}{1 + C' \cos \beta'} = A' f(\beta) \quad (\text{A-26})$$

The maximum damping is a function of the loss factor of the damping material as well as of the relative dimensions of the two layers of the treatment. At non-optimum values of  $\Gamma$ , the damping factor is less than  $\eta_{\text{max}}$ :

$$\frac{\eta}{\eta_{\text{max}}} = \frac{1}{1 + \frac{(\Gamma/\Gamma_{\text{opt}} - 1)^2}{2(1 + C' \cos \beta')(\Gamma/\Gamma_{\text{opt}})}} \quad (\text{A-27})$$

This function is geometrically symmetric with respect to  $\Gamma/\Gamma_{\text{opt}}$ .

#### Graphs of General Functions Used in Damping Calculations

The general results of the analysis of damping by constrained damping layers are expressed by Eqs (A-25), (A-26), and (A-27) in terms of coefficients  $A'$ ,  $C'$  and  $\beta'$  which are functions of the physical properties of the layers making up the treatment. The important parameter is  $k_3$ , the relative stiffness of the outer layer. Other parameters are  $h_2$  and  $h_3$ , the latter being related to  $k_3$  through

$k_3 = e_3 h_3$ . To enable rapid calculations of damping and the later development of design charts, graphs are needed for the various coefficients in terms of the basic physical quantities.

For making our calculations, we have chosen values of  $e_3$  of  $1/3$ , 1 and 3. If both the foil and the plate are made of the same material, then  $e_3 = 1$ . If one is of a light metal and the other a heavy metal, then the ratio of the Young's moduli will be close to  $1/3$  or 3, depending on which one is on top. It is unlikely that the damping layer will have a thickness greater than a third that of the plate, so values of  $h_2 = 0, 0.1, 0.2$  and  $0.3$  have been selected.

Figures 5 to 7 give the dependence of  $\Gamma_{\text{opt}}$  on  $e_3$ ,  $h_2$  and  $k_3$ . Actually, the three sets of curves are very similar for  $k_3 < 0.1$ , only diverging for the higher values. Within the accuracy of many calculations, Fig 6 can be used for all values of  $e_3$ , and the curve for  $h_2 = 0.1$  is representative of all values of  $h_2$ . The function of  $\cos \beta'$  needed to complete the calculation of  $\Gamma_{\text{opt}}$  is given in Fig 12 as a function of  $\beta$ .

The maximum loss factor is a function of  $\beta$  and of the parameters  $e_3$ ,  $h_2$  and  $k_3$ . Figures 8 to 10 show its dependence on the latter group. In this case variations of both  $h_2$  and  $e_3$  are important. The function  $f(\beta)$  is a function of  $C'$  as well as of  $\beta$ . As shown in Fig 11,  $C'$  is a slowly varying function of  $A'$ , having values between 1.0 and 1.4. (That  $C'$  should be related to  $A'$  was shown both analytically and empirically before Fig 11 was drawn.) We may therefore relate all functions of  $C'$  to functions of  $A'$ , choosing three values of  $C'$  as representative of the full range of  $A'$  as follows:

$0 < A' < .3$	$C' = 1.02$
$.3 < A' < .6$	$C' = 1.10$
$.6 < A' < 1$	$C' = 1.26$

In Fig 12, the various functions of  $\beta'$  are plotted against  $\beta$ , using the three representative values of  $C'$  where required. It is clear that the variations caused by  $A'$ ; i.e., by  $k_3$ ,  $e_3$  and  $h_2$ , are small. As the loss factor of the material is usually not known with any great precision, it is reasonable to neglect the small effect of these other factors.

#### Dependence on $\Gamma/\Gamma_{opt}$ .

The ratio of the damping for any value of  $\Gamma$  to the maximum achievable at  $\Gamma_{opt}$  is primarily a function of the ratio of  $\Gamma$  to  $\Gamma_{opt}$  and secondarily a function of  $\beta$  and the dimensions of the layers. We may neglect the variation with layer dimensions and consider only the effect of  $\beta$ . Even this is a slow variation and only a few typical values of  $(1 + C' \cos \beta')$  need be used in Eq (A-27). The values used are the following:

$0 < \beta < .6$	$(1 + C' \cos \beta') = 2.00$
$.6 < \beta < 1.4$	$= 1.75$
$1.4 < \beta < 3$	$= 1.50$

Figure 13 shows how  $\eta/\eta_{max}$  varies with  $\Gamma/\Gamma_{opt}$  for these three ranges of  $\beta$ . Because of the geometric symmetry of the function one set of curves applies whether the shear parameter be greater or less than optimum. As the variation with  $\beta$  is relatively

small, and as  $\beta$  is most often between 0.6 and 1.4 for typical damping materials, it will usually be within the desired accuracy simply to use the middle curve. To within  $\pm 10\%$ :

$$\frac{\eta}{\eta_{\max}} = \frac{3.65 (\Gamma/\Gamma_{\text{opt}})}{(1 + \Gamma/\Gamma_{\text{opt}})^2} \quad (\text{A-28})$$

no matter what the value of the loss factor  $\beta$ .

### The Shear Parameter

The shear parameter defined by Eq (A-8) is a function of the shear modulus and thickness of the middle layer, of the stiffness of the base plate, and of the wave-number of the vibrations. The shear modulus and wave-number are both functions of frequency. The former is a property of the viscous material, while the wave-number is related to the frequency through the velocity of the flexural waves by:

$$p = \frac{\omega}{c_B} \quad (\text{A-29})$$

The flexural velocity is influenced by the added stiffness and mass of the damping layer. If we assume that this is a second-order correction, then we may estimate  $c_B$  from the properties of the undamped plate. The formula for the flexural wave velocity for an undamped plate is:

$$c_B = \sqrt{\frac{\alpha H_1 c_0}{\sqrt{12}}} \quad (\text{A-30})$$

where  $c_o$  is the usual velocity of sound in the plate. Using this relationship, the expression for the shear parameter becomes:

$$\Gamma \doteq \frac{c_o}{2\pi\sqrt{12}f} \frac{1}{h_2} \frac{G_2}{K_1} \quad (\text{A-31})$$

For most metal plates, the longitudinal velocity of sound is closely 17,000 ft/sec, so that the shear parameter can be calculated from:

$$\Gamma \doteq \frac{790}{fh_2} \frac{G_2}{K_1} \doteq \frac{720}{f} \frac{G_2}{E_1 h_2} \quad (\text{A-32})$$

where  $E_1$  is the Young's modulus of the plate material and a correction has been made for Poisson's ratio.

The value of Young's modulus is about  $7.3 \times 10^{11}$  dynes/cm<sup>2</sup> for aluminum and  $20 \times 10^{11}$  dynes/cm<sup>2</sup> for steel. The shear modulus of visco-elastic materials varies from  $10^6$  dynes/cm<sup>2</sup> when very soft to about  $10^{10}$  dynes/cm<sup>2</sup> when hard. The thickness of the damping layer could be as small as  $10^{-4}$  ft or as large as  $2 \times 10^{-2}$  ft. It follows that possible values of  $\Gamma$  could vary by as much as  $10^6$ , at a given frequency. However, significant damping can only be achieved in the range:  $0.002 < \Gamma < 2$ . By assuming values of Young's modulus appropriate to the plate material, we can plot  $\Gamma$  as a function of the shear modulus of the damping material, with frequency and damping layer thickness as parameters. Fig 14 is such a plot to be used to estimate the shear parameter when the base plate is made of aluminum, while Fig 15 applies to steel plates.



### Use of the Charts

The charts presented in Figs 5 through 15 can be used to calculate the shear damping of a given constrained damping-layer treatment. Thus, knowing the thicknesses of the foil and damping layer relative to that of the base plate, one can readily compute  $h_3$  and  $h_2$ . The materials of the plate and foil determine  $e_3$  and thus  $k_3$ . Going into the appropriate chart among Figs 5 through 7, we obtain  $\Gamma_{\text{opt}}/\cos \beta'$ . In the same way, Figs 8 through 10 give  $A' = \eta_{\text{max}}/f(\beta)$ . These two results are independent of the damping material and therefore of temperature and frequency. Given the properties of the damping material, the appropriate values of the loss factor can be tabulated for each combination of frequency and temperature. Fig 12 gives both  $\cos \beta'$  and  $f(\beta)$  for each value of  $\beta$ . One can then calculate  $\eta_{\text{max}}$  and  $\Gamma_{\text{opt}}$  for each combination of frequency and temperature. It then only remains to determine the actual values of  $\Gamma$  from Figs 14 or 15 and to calculate the reduction in  $\eta$  caused by non-optimum values of  $\Gamma$ , using Fig 13.

As an example of the use of these charts let us carry through the calculation of the damping curve for a damping treatment consisting of 1/32 inch polyisobutylene and 1/8 inch steel on a 1-inch steel plate at room temperature. As the constraining layer is the same material as the plate,  $e_3 = 1$ . From geometry:

$$h_2 = \frac{1}{32} = .030$$

$$k_3 = h_3 = \frac{1}{8} = .125$$

From Fig. 6:  $\frac{\Gamma_{\text{opt}}}{\cos \beta'} = \frac{1}{\sqrt{D}} = .091$

From Fig. 9:  $\frac{\eta_{\text{max}}}{f(\beta)} = A' = .195$

From Fig 11:  $c' = 1.02$

From references 8 and 9 (or Fig 14 of Ref 7) concerning the dynamic mechanical properties of polyisobutylene, we can determine the shear modulus and loss factor for several typical frequencies at room temperature:

<u>Frequency</u>	<u>G</u>	<u><math>\beta</math></u>
30 cps	$4.2 \times 10^6$ dynes/cm <sup>2</sup>	0.6
100 cps	6.0	0.85
300 cps	9.2	1.2
1000 cps	16.5	1.5

From Fig 12, we obtain  $\cos \beta'$  and  $f(\beta)$  and thus calculate  $\Gamma_{\text{opt}}$  and  $\eta_{\text{max}}$ :

<u>Frequency</u>	<u><math>\cos \beta'</math></u>	<u><math>\Gamma_{\text{opt}}</math></u>	<u><math>f(\beta)</math></u>	<u><math>\eta_{\text{max}}</math></u>
30 cps	0.85	.077	0.28	.055
100 cps	0.76	.069	0.37	.072
300 cps	0.65	.059	0.47	.092
1000 cps	0.55	.050	0.53	.104

From Fig 15, the shear parameter can be read and the ratio to optimum calculated:

<u>Frequency</u>	<u><math>fH_2</math></u>	<u><math>G_2</math></u>	<u><math>\Gamma</math></u>	<u><math>\Gamma/\Gamma_{opt}</math></u>	<u><math>\Gamma_{opt}/\Gamma</math></u>
30 cps	0.9	$4.2 \times 10^6$	.021	0.27	3.7
100 cps	3	6.0	.0085	0.123	8.1
300 cps	9	9.2	.0044	0.075	13.5
1000 cps	30	16.5	.0024	0.048	21

Finally, the reduction in  $\eta$  caused by non-optimum  $\Gamma$  is obtained from Fig 13 and  $\eta$  then computed:

<u>Frequency</u>	<u><math>\eta/\eta_{max}</math></u>	<u><math>\eta</math></u>
30 cps	0.64	.035
100 cps	0.36	.026
300 cps	0.225	.021
1000 cps	0.14	.0145

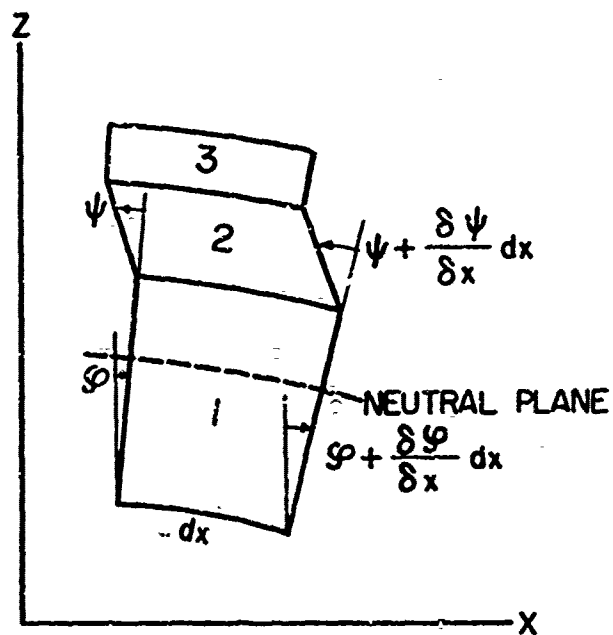


FIG. A-1 ELEMENT OF A THREE-LAYER PLATE IN FLEXURAL VIBRATION, SHOWING THE FLEXURAL ANGLE  $\varphi$  AND SHEAR ANGLE  $\psi$ .

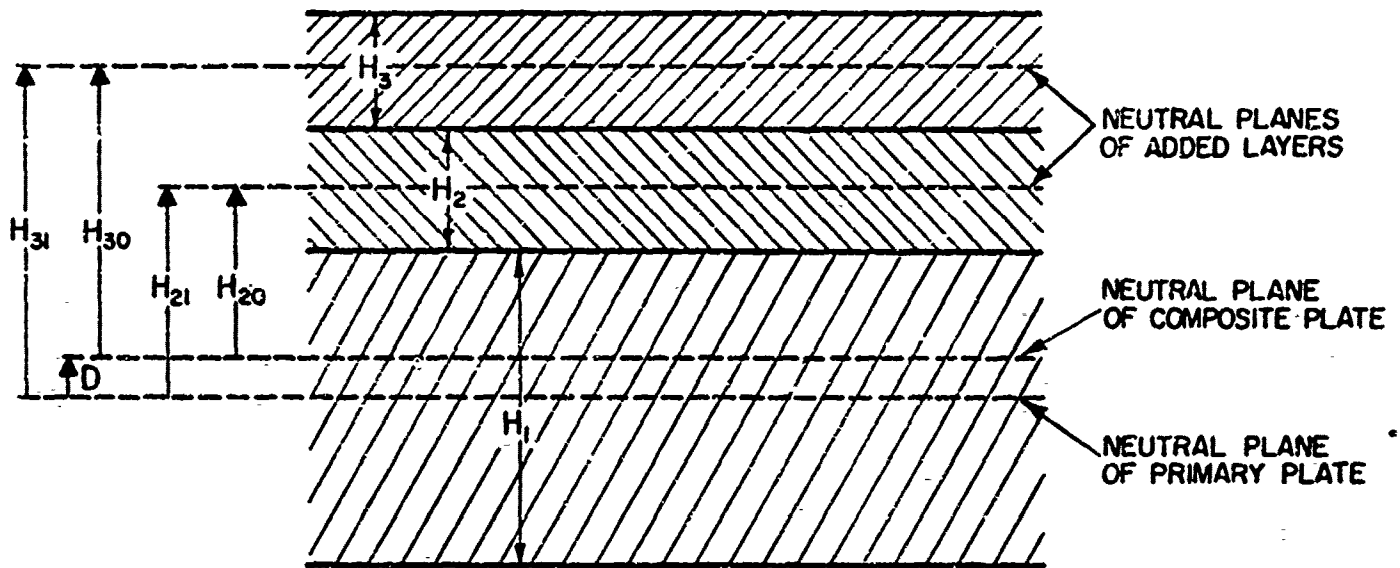


FIG.A-2 DIMENSIONS USED IN ANALYSIS OF  
A THREE-LAYER PLATE IN FLEXURE.

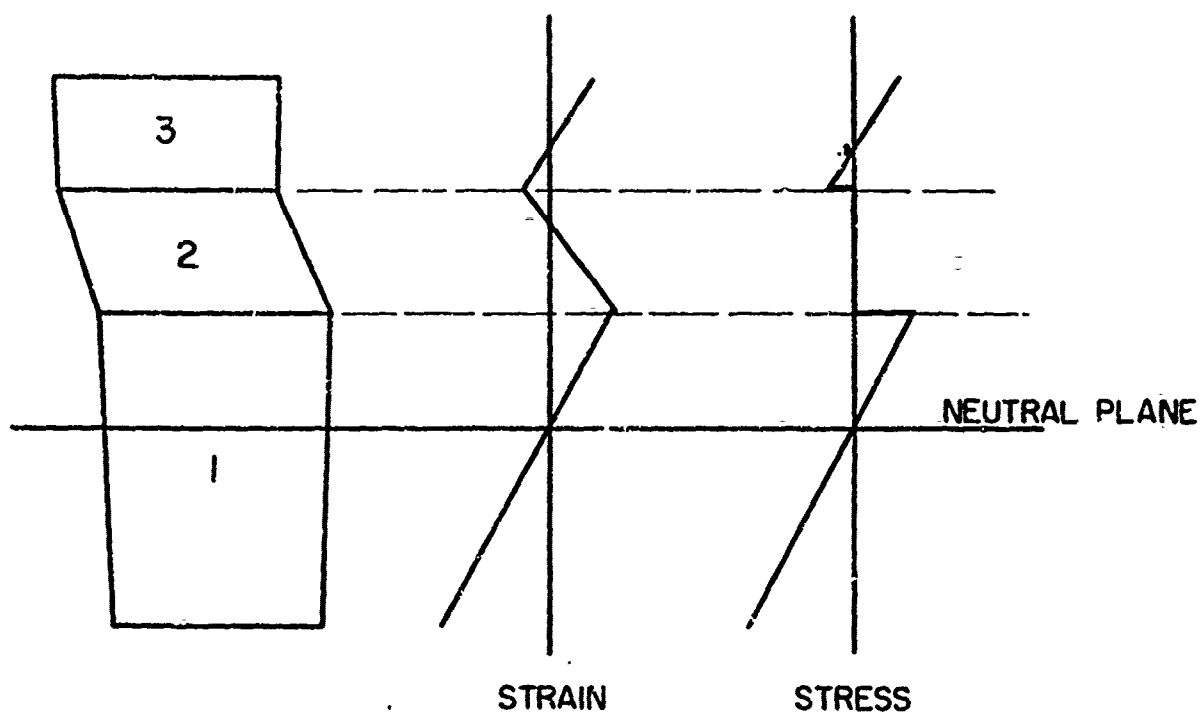


FIG.A-3 EXTENSIONAL STRAIN AND STRESS DISTRIBUTIONS FOR THREE-LAYER PLATE ELEMENT IN FLEXURE.

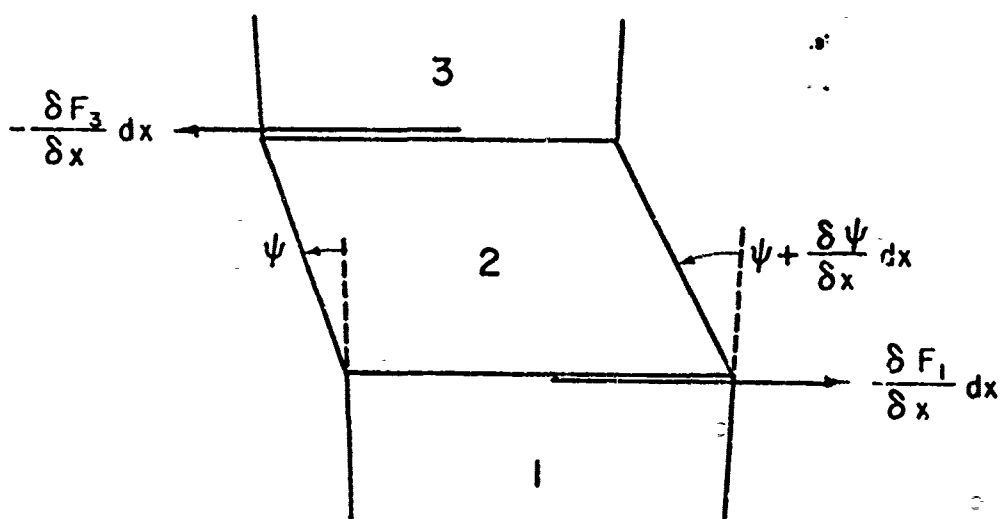


FIG. A-4 SHEAR FORCE ON MIDDLE LAYER.

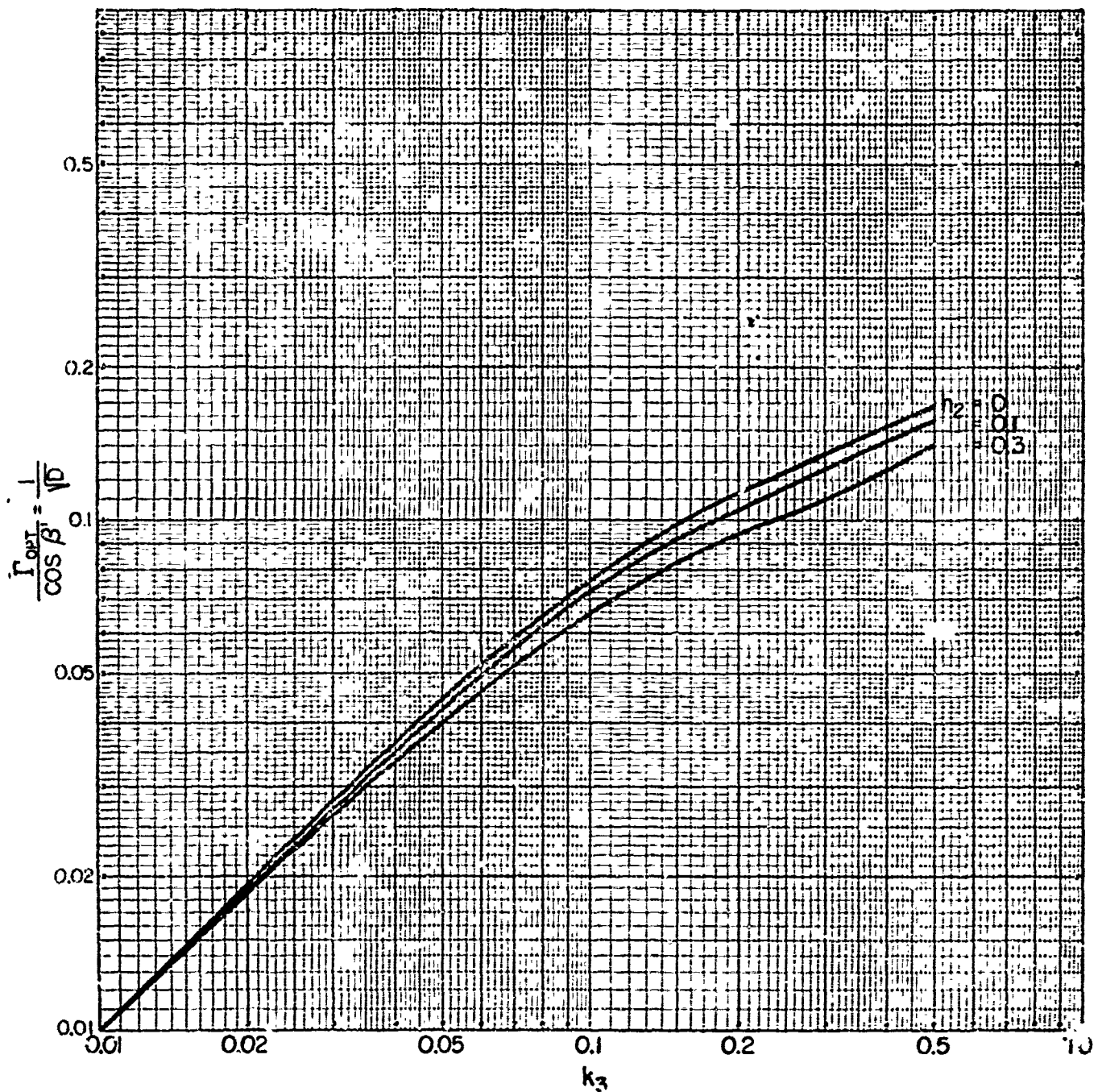
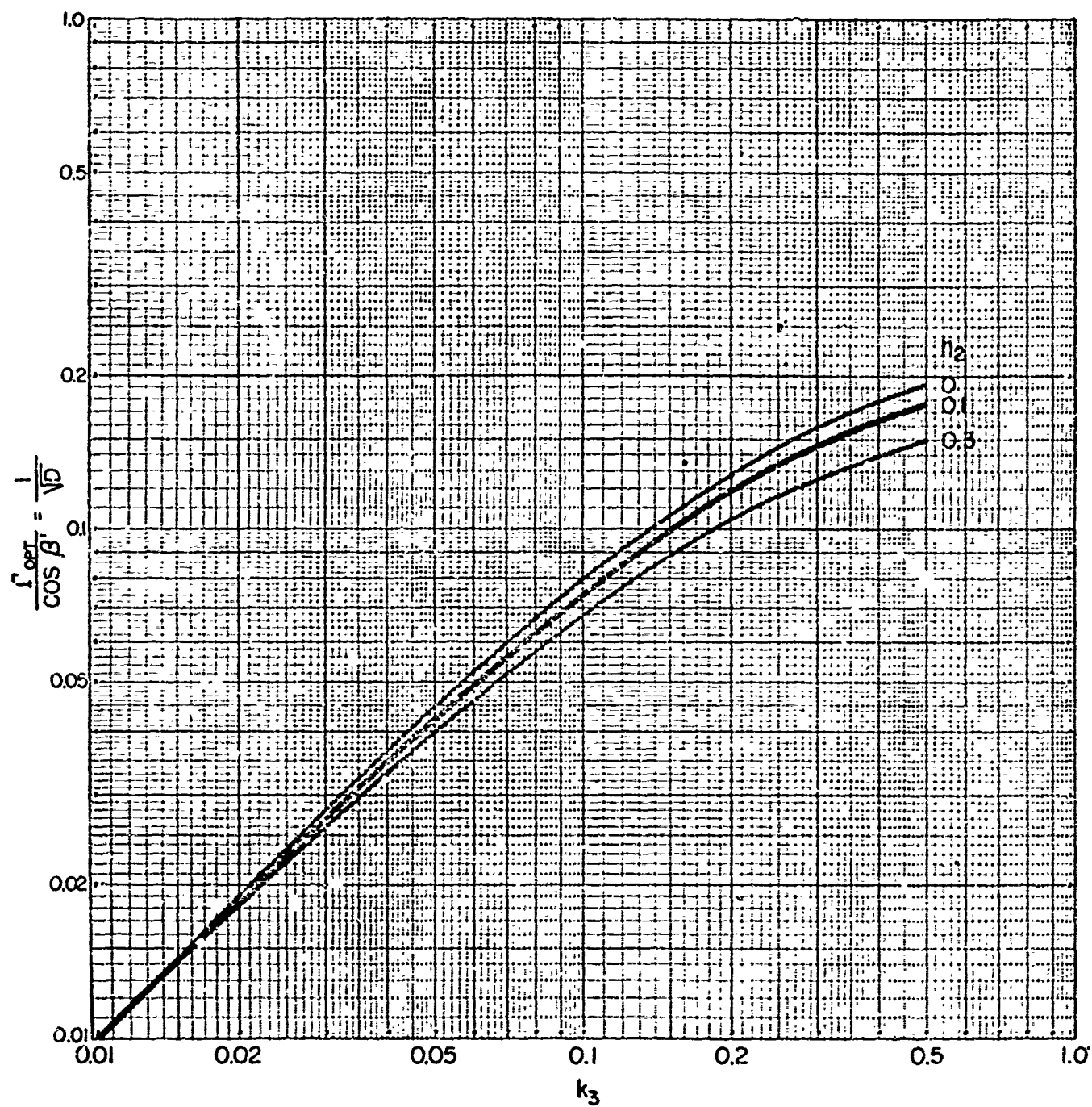
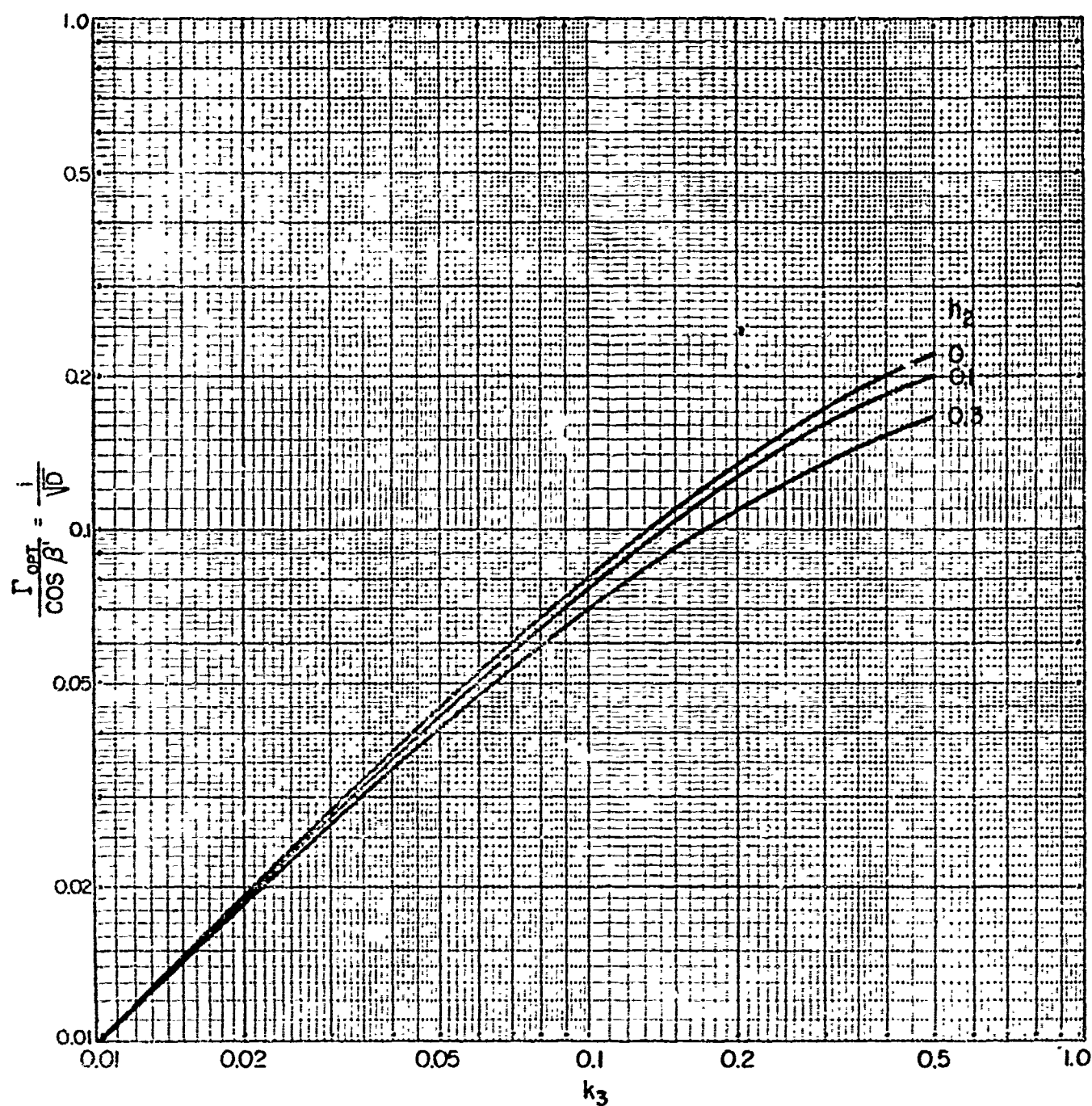
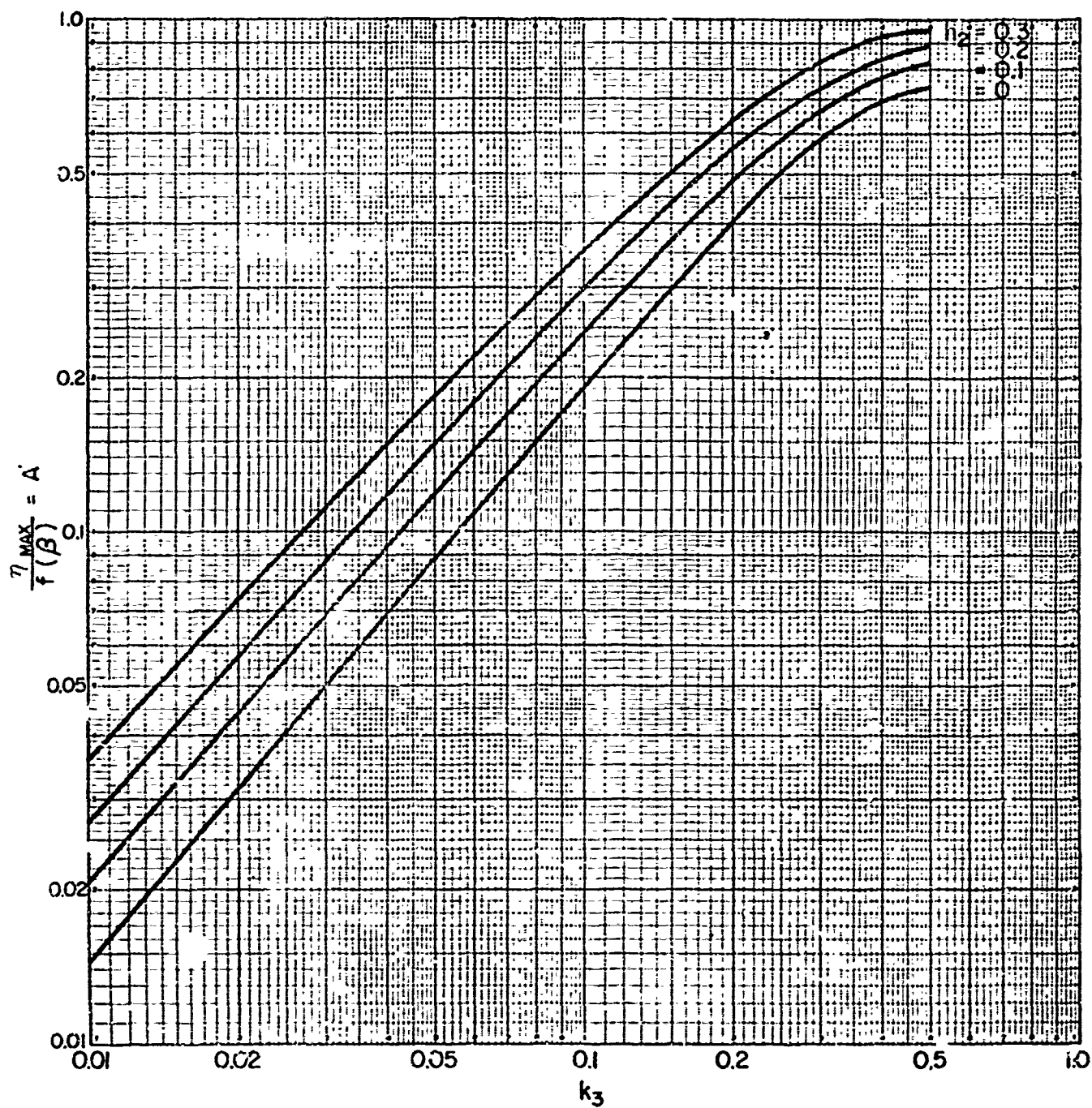


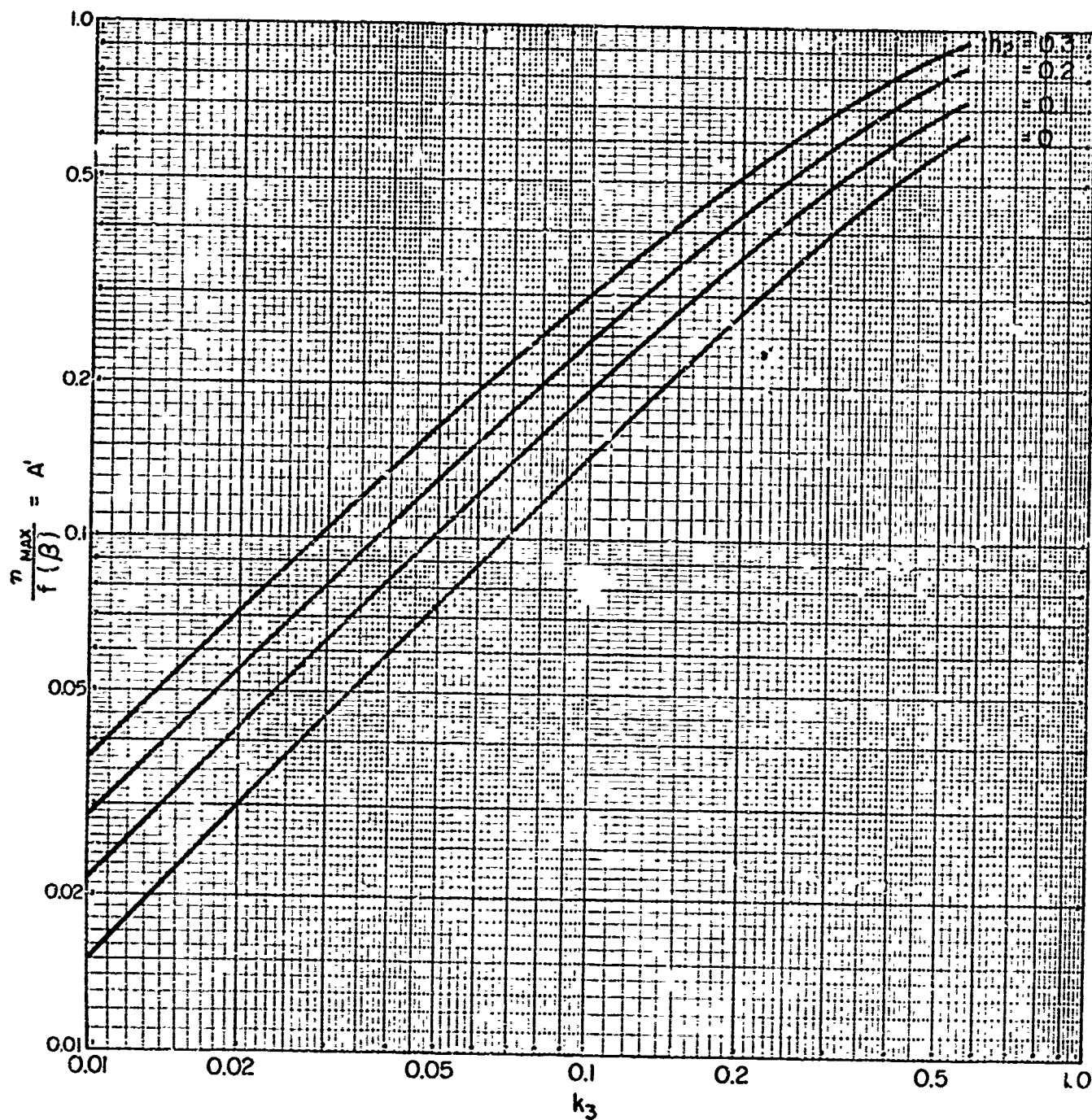
FIG. A-5  $\Gamma_{OPT}$  FOR  $e_3 = \frac{1}{3}$



FIG. A-6  $\Gamma_{OPT}$  FOR  $e_3 = 1$

FIG. A-7  $\Gamma_{OPT}$  FOR  $e_3 = 3$

FIG. A-8  $\eta_{MAX}/f(\beta)$  FOR  $e_3 = \frac{1}{3}$

FIG. A-9  $\eta_{MAX}/f(\beta)$  FOR  $e_3 = 1$

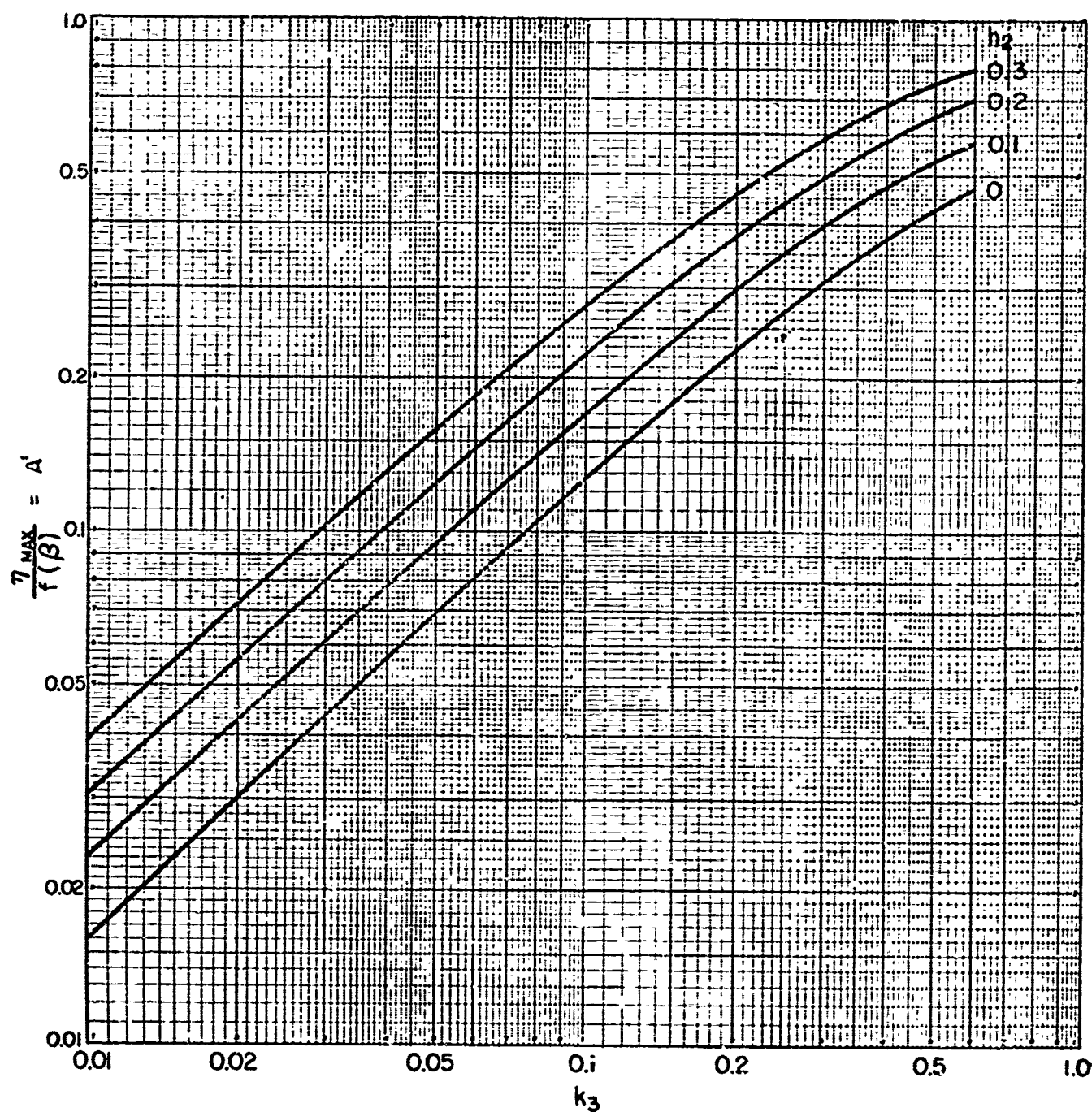
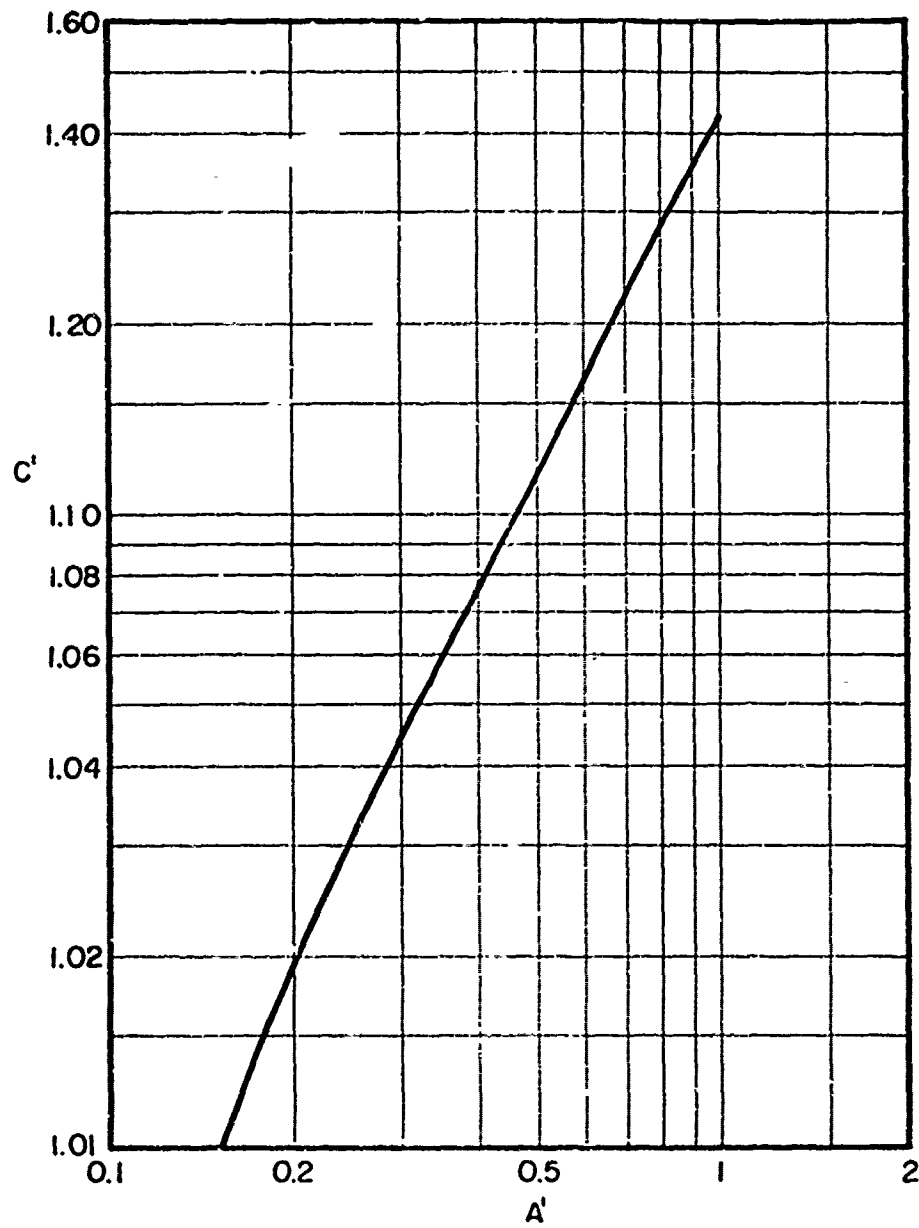
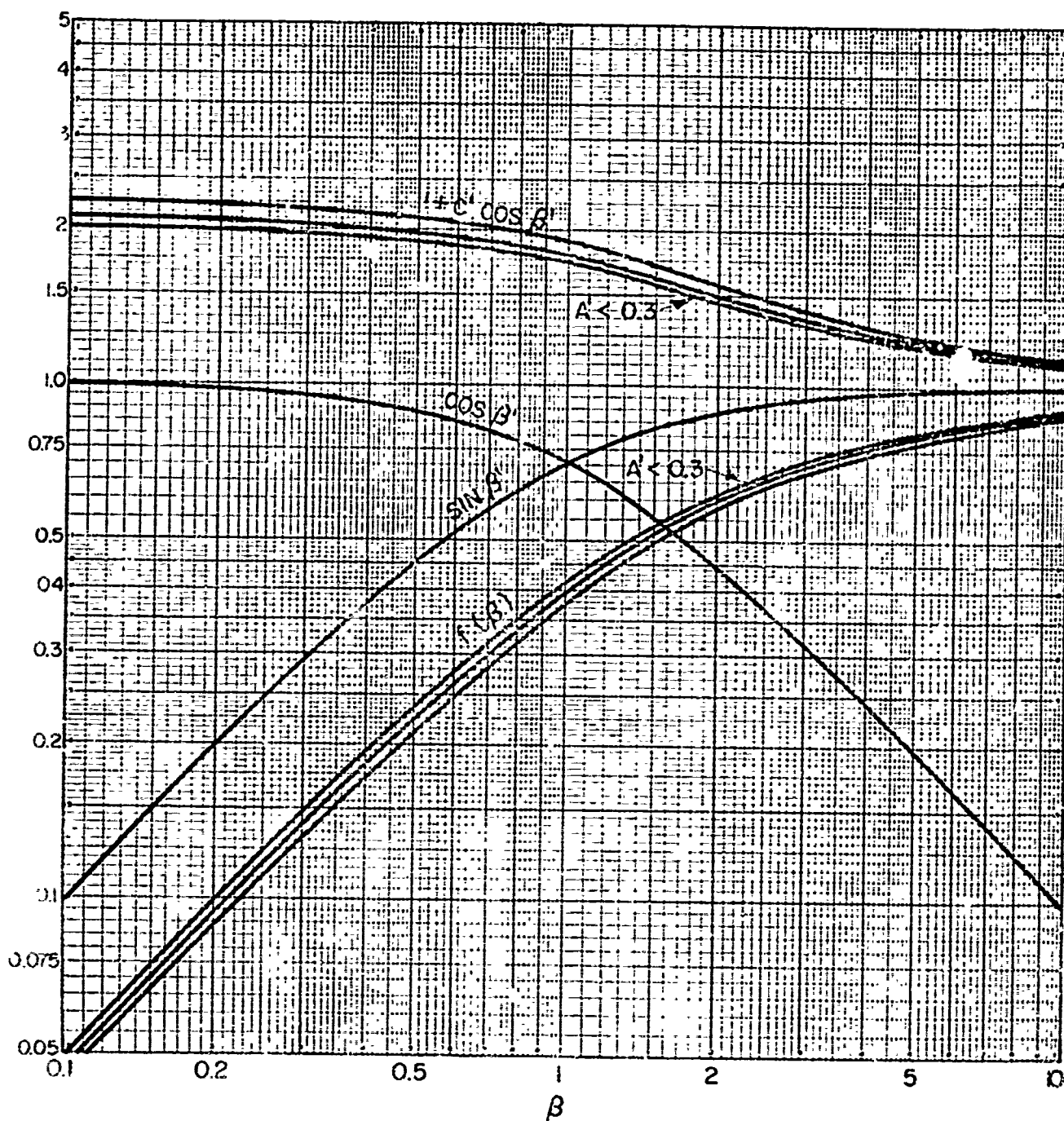


FIG. A-10  $\frac{\eta_{MAX}}{f(\beta)}$  FOR  $e_3 = 3$

FIG. A-II  $C'$  AS A FUNCTION OF  $A'$



FIG. A-12 FUNCTIONS OF  $\beta$

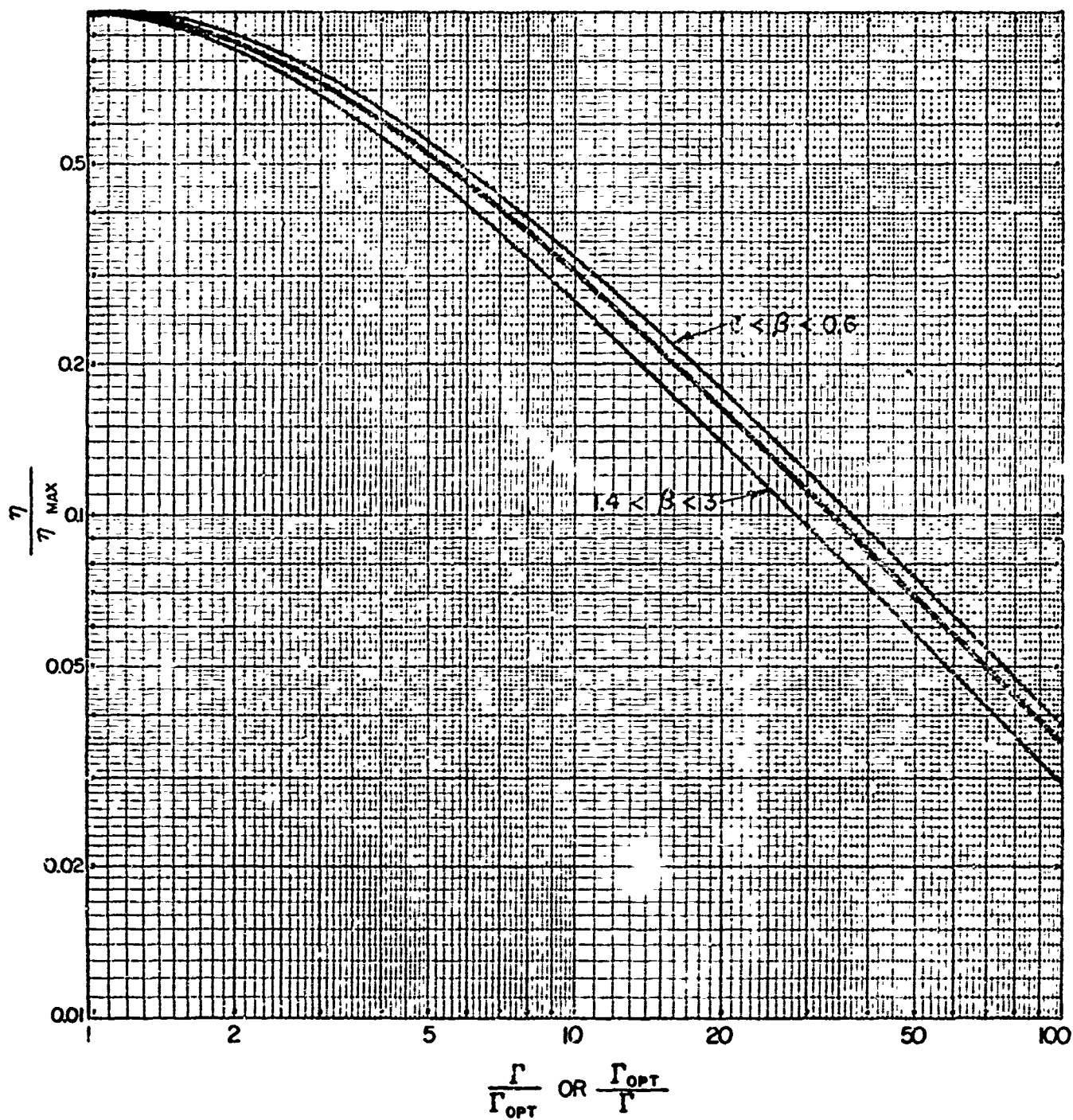


FIG. A-13  $\frac{\eta}{\eta_{\text{MAX}}}$  VS  $\frac{\Gamma}{\Gamma_{\text{OPT}}}$



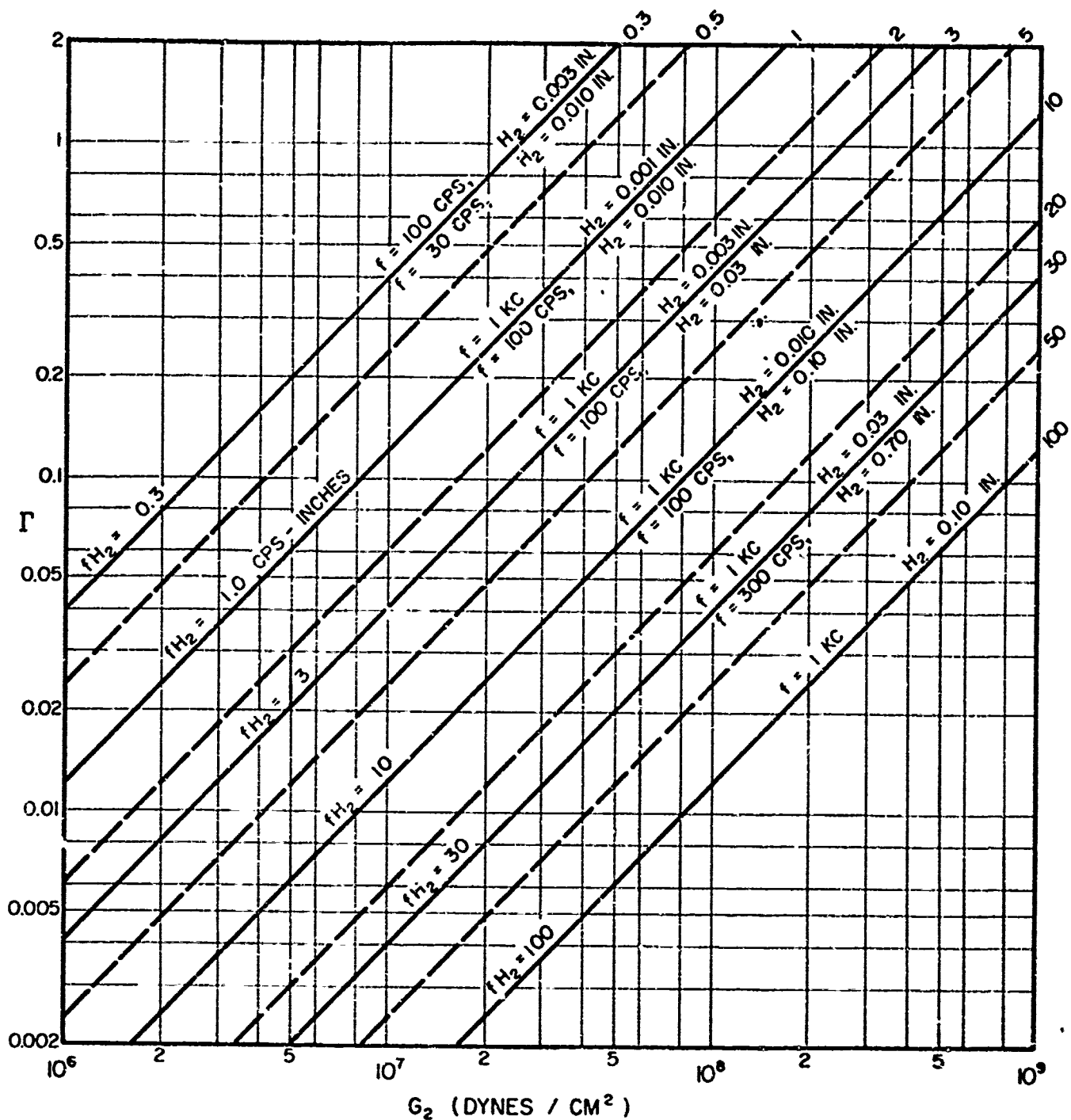


FIG. A-14 SHEAR PARAMETER, ON ALUMINUM PLATES

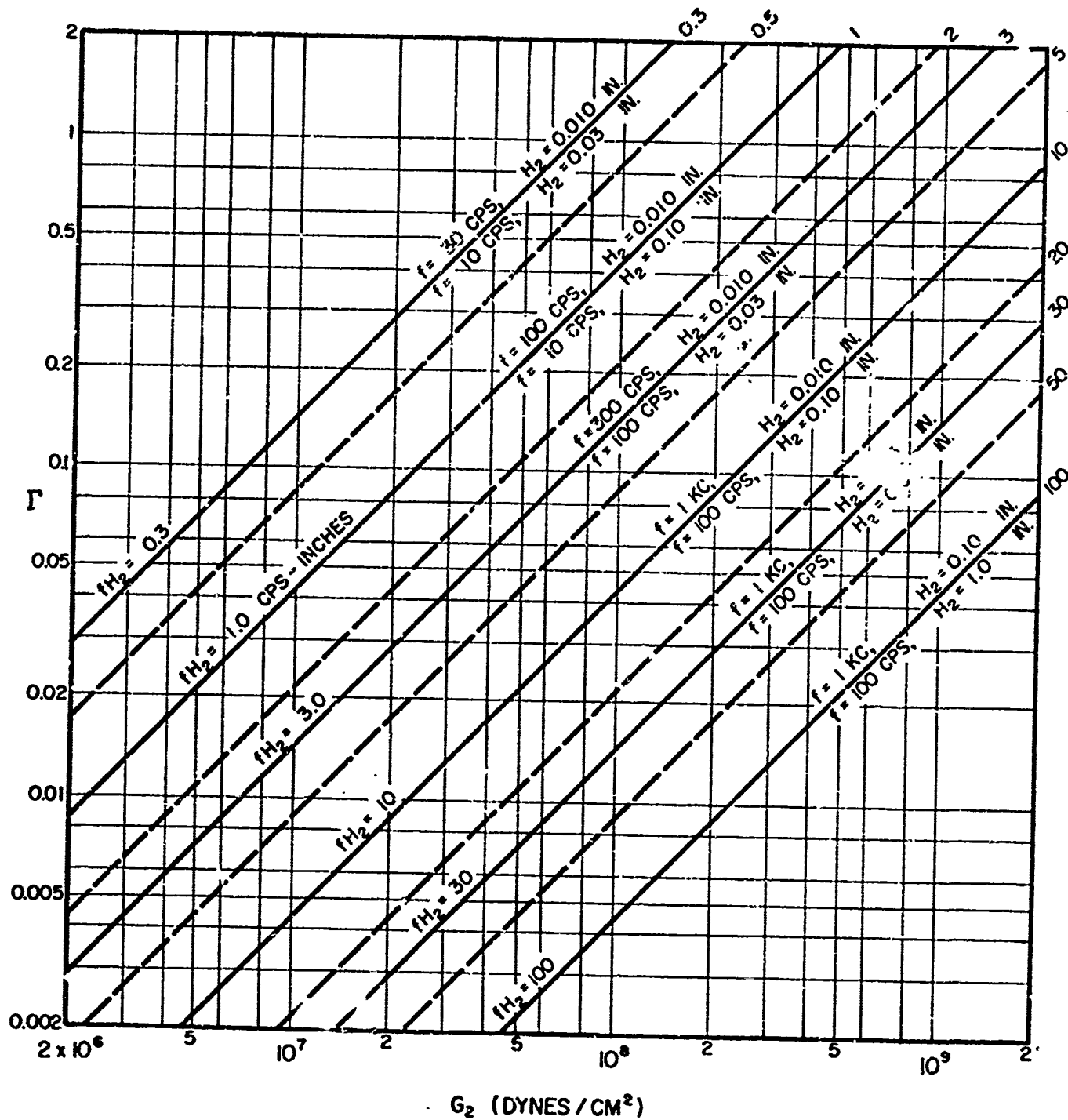


FIG. A-15 SHEAR PARAMETER, ON STEEL PLATES

## APPENDIX B

### MEASURES OF DAMPING EFFECTIVENESS

#### Loss Factor, $\eta$

The velocity of propagation of flexural waves on an undamped and unloaded plate is:

$$c_B = \sqrt{\omega} \sqrt[4]{\frac{B}{m}} \quad (B-1)$$

where  $B$  is the flexural rigidity per unit width and  $m$  is the mass per unit area. As the velocity is dependent on the frequency, flexural waves are said to be dispersive. We may treat the damping caused by the internal friction of the materials composing the plate by defining a complex flexural rigidity

$$B^* = B' + jB'' = B(1 + j\eta_B) \quad (B-2)$$

where  $\eta_B$  is the loss factor. If there is damping caused by radiation or other wave effects, then the mass is expressed as a complex quantity:

$$m^* = m' - jm'' = m(1 - j\eta_m) \quad (B-3)$$

If both types of damping occur, then:

$$\left(\frac{B}{m}\right)^* = \left(\frac{B}{m}\right)_e (1 + j\eta) \quad (B-4)$$

where the effective ratio of the rigidity to mass is:

$$\left(\frac{B}{m}\right)_e = \left(\frac{B}{m}\right) \frac{1 - \eta_B \eta_m}{1 + \eta_m^2} \quad (B-5)$$

and:

$$\eta = \frac{\eta_B + \eta_m}{1 - \eta_B \eta_m} \quad (B-6)$$

As the individual loss factors are seldom greater than 0.2, the total effective loss for the plate is closely equal to their sum. If the total loss factor is less than 0.4, then the expression for the complex velocity of propagation is  $v$  in accurately:

$$c_B^* \doteq \sqrt{\omega} \sqrt[4]{\frac{B}{m}} \left(1 + j \frac{\eta}{4}\right) \quad (B-7)$$

The loss factor,  $\eta$ , defined by Eq (B-4) is a common measure of the effectiveness of damping.

#### Figure-of-Merit, Q

The frequency response of a finite plate contains many resonances. Another measure of damping is the figure-of-merit or Q of the resonances, defined by:

$$Q \equiv \frac{f_0}{\Delta f} \quad (B-8)$$

where  $\Delta f$  is the bandwidth between the two half-power points of the resonance. The  $Q$  of a resonance is equal to the reciprocal of the loss factor at the frequency of the resonance:

$$Q \doteq \frac{1}{\eta} \quad (\text{B-9})$$

provided the damping factor is not greater than about 0.2.

#### Fraction of Critical Damping

In many vibratory problems in mechanics the free motion is governed by a linear, second-order differential equation of the form:

$$m\ddot{x} + c\dot{x} + kx = 0 \quad (\text{B-10})$$

Only if the damping coefficient,  $c$ , is less than a critical value,

$$c_c = 2\sqrt{mk} \quad (\text{B-11})$$

is the motion oscillatory. The ratio of the damping coefficient,  $c$ , to the critical damping coefficient is then a measure of the damping effectiveness.

For low values of damping the shapes of the resonance curves of a plate resonance and a lumped-parameter system whose motion is governed by Eq (B-10) are similar. Therefore a value of  $c/c_c$  can be attributed to a given value of the loss factor  $\eta$ . The  $Q$  of a simple mechanical system is given by:

$$Q \doteq \frac{1}{2} \frac{c_o}{c} \quad (\text{B-12})$$

whence the equivalent fractional critical damping of a plate vibration is:

$$\frac{c}{c_c} \doteq \frac{1}{2} \eta \quad (\text{B-13})$$

Equation (B-13) can be used as the definition of the fractional critical damping for flexural vibrations of plates.

#### Space and Time Decay Rates

Damping may be measured by observing the time-rate-of-decay of a vibration after the driving force is removed, or by measuring the spatial decay when a steady-state force is applied at one place. For values of  $\eta$  less than 0.2, these decay rates can be expressed in terms of  $\eta$  by the following:

$$D_t = 27.3\eta f_0 \text{ (db/sec)} \quad (\text{B-14})$$

$$D_\lambda = 13.6\eta \text{ (db/wavelength)} \quad (\text{B-15})$$

$$D_\ell = 13.6\eta/\lambda \text{ (db/unit length)} \quad (\text{B-16})$$

#### Significance of the Loss Factor

Although any of the many ways of describing damping may be used for small amounts of damping, it is best to express analytical results in terms of a parameter that has meaning for highly damped as well as lightly damped structures and which applies to distributed systems as well as lumped-parameter systems. The loss factor,  $\eta$ , defined by Eq (B-4) is quite general. For this reason, we shall express all of our results in terms of  $\eta$ , and the other quantities may then be estimated from Eqs (B-9), (B-13), (B-14), (B-15), and (B-16).